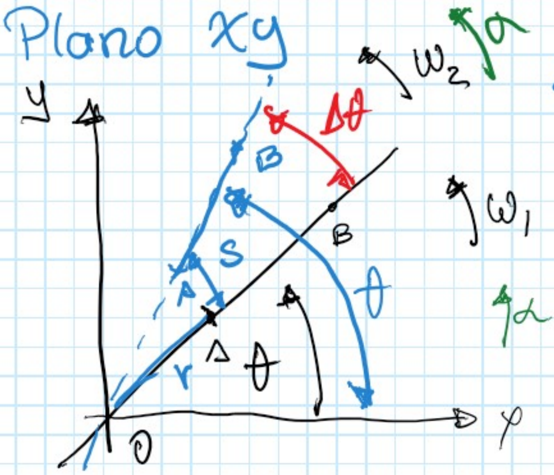
Movimiento Angular

Wednesday, October 24, 2018 11:03 AM

Descripción del movimiento angular de un segmento de recta

segmento de recta = cuerpo extenso sencillo

Plano xy



posición angular = θ

$\measuredangle$ ángulo

Δt . instante (t): θ rad

$\Delta\theta = \theta_{t_2} - \theta_{t_1}$ $\Delta t = t_2 - t_1$ $\Delta\theta = \theta_f - \theta_0$

velocidad angular

$$\omega_m = \frac{\Delta\theta}{\Delta t}, \text{ rad/s}$$

$\Delta\theta = \frac{\Delta s}{r}$ $\Delta s = \Delta\theta \cdot r$ derivada

$v = \omega r$

$$\omega_m = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} = \dot{\theta}$$

$\theta = f(t)$

$\omega = g(t)$

$\alpha = h(t)$

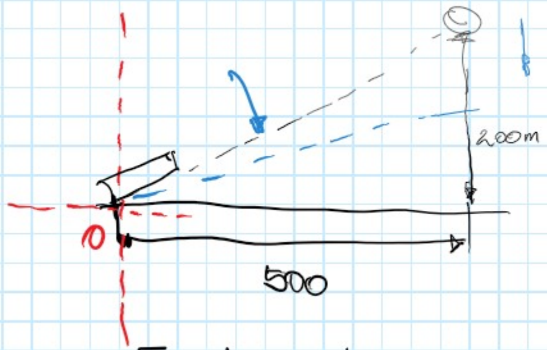
Aceleración angular (α)

$$\alpha_m = \frac{\Delta\omega}{\Delta t}, \text{ rad/s}^2$$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \dot{\omega} = \frac{d^2\theta}{dt^2} = \ddot{\theta}$$

desde una altura de 200m sobre el suelo se abandona un pequeño cuerpo y su movimiento es seguido por un telescopio como se indica en la figura

$t = 2\,s$ = velocidad angular del telescopio



C : MRUV

$y = 200 + \frac{1}{2} g t^2$

$y = 200 - 4.9t^2$ m

$v = -9.8t$ m/s

$a_y = -9.8$ m/s²

T = Vel angular

$\theta(2) = 0.3462$ rad.

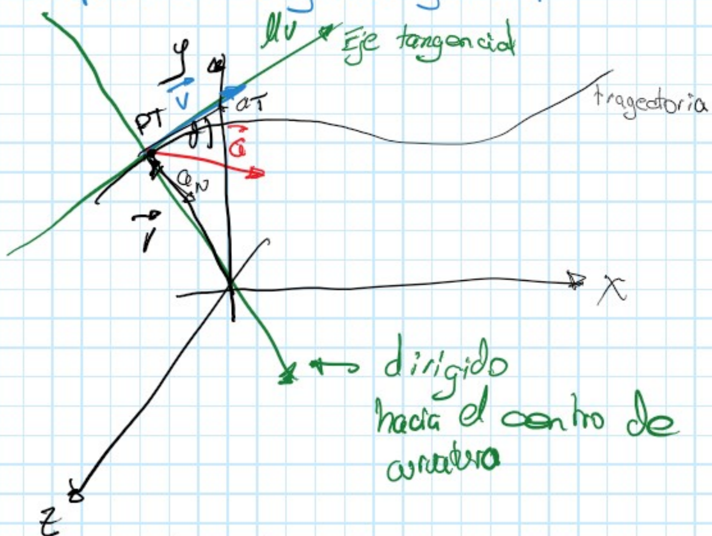
$\tan\theta = \frac{y}{500}$

$\dot\theta = \frac{1}{500} \cdot \frac{\dot y}{\sec^2(\theta)}$

$\sec^2\theta \cdot \dot\theta = \frac{\dot y}{500}$

Coordenadas naturales

componentes tangencial y normal



eje tangencial

plano osculador formado entre la velocidad y la aceleración

$\vec V = \vec V_N + V_T$

no hay componente normal es perpendicular

$\vec v = v(\vec u_T)$, $\vec u_T = u_v$

$\vec a = \vec a_N + \vec a_T$

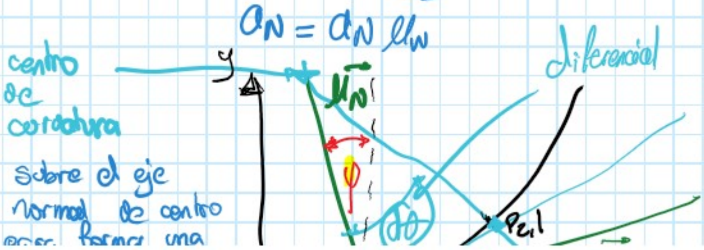
aceleración normal o centrípeta

$\vec a_T = \pm a_T \vec u_T$

$\vec a_T = \vec a_v = \frac{\vec a \cdot \vec v}{v^2} \vec v$

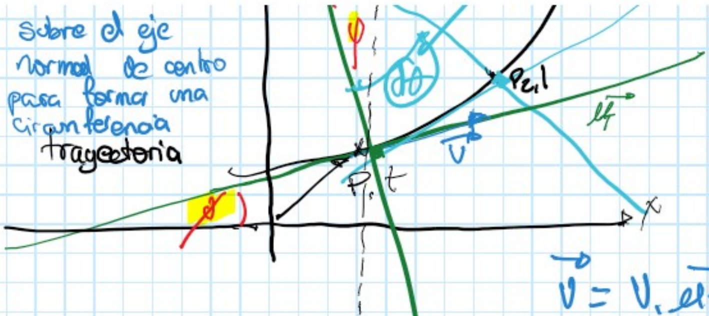
$\vec a = \vec a_N + \vec a_T$

$\vec a_N = a_N \vec u_N$



Sobre el eje normal de centro pasa formar una circunferencia

trayectoria



$$\vec{v} = v \cdot \vec{u}_T$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d(v \cdot \vec{u}_T)}{dt} = \frac{dv}{dt} u_T + v \cdot \frac{du_T}{dt}$$

$$\vec{u}_T = \cos\varphi\, i + \operatorname{sen}\varphi\, j$$

$$\vec{u}_N = -\operatorname{sen}\varphi\, i + \cos\varphi\, j$$

radio de curvatura

$\vec{\rho}$

$$\frac{d\vec{u}_T}{dt} = -\operatorname{sen}\varphi \cdot \frac{d\varphi}{dt}\vec{i} + \cos\varphi \frac{d\varphi}{dt}\vec{j} = u_N \frac{d\varphi}{dt}$$

$$d\varphi = \frac{ds}{\rho}$$

$$\frac{d\varphi}{dt}\left(-\operatorname{sen}\varphi\, i + \cos\varphi\, j\right)$$

$$\frac{d\varphi}{dt} = \frac{ds}{\rho\, dt} = \frac{v}{\rho}$$

$$\frac{v}{\rho} \cdot u_N$$

$$\vec{a} = \frac{dv}{dt}\vec{u}_T + v \cdot \frac{v}{\rho}\vec{u}_N$$

$$\vec{a} = \frac{dv}{dt}\vec{u}_T + \frac{v^2}{\rho}\vec{u}_N$$

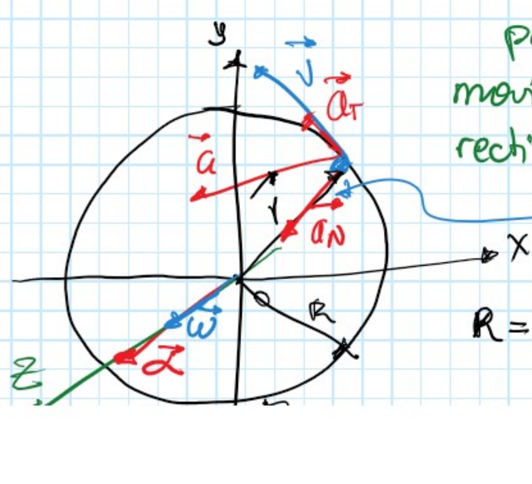
$$\vec{a} = \pm a_T \vec{u}_T + a_N \vec{u}_N$$

$$a_T = \frac{dv}{dt} = \frac{d^2 s}{dt^2} = \ddot{s}$$

$$a_N = \frac{v^2}{\rho}$$

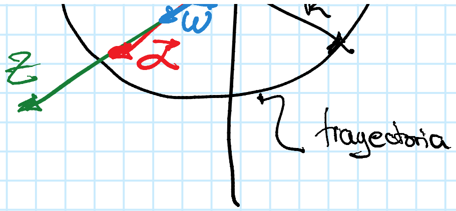
radio de curvatura tiende al infinito

para movimientos rectilíneos



radio vector describe el círculo.

$$R = |\vec{r}| = cte$$



$R = |r| = cte$

$x^2 + y^2 = r^2$ ← ecuación de la trayectoria

$\theta = \frac{S}{R} \Rightarrow S = \theta . R$

$\Rightarrow dS = d\theta R$

$V = \omega R$

$\Rightarrow dv = d\omega R$

$a_T = \alpha . R$

$\vec{v} = \vec{\omega} \times \vec{r}$

$\vec{a}_T = \vec{\alpha} \times \vec{r}$

$\vec{a}_N = \vec{\omega} \times \vec{V}$

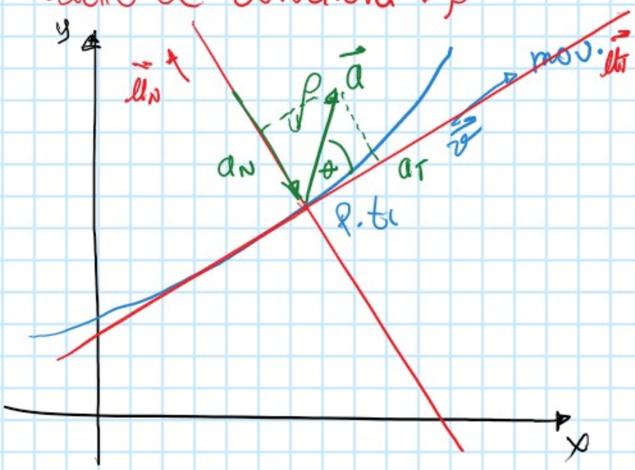
$a_N = \frac{v^2}{R} = \frac{\omega^2 . R^2}{R} = \omega^2 R$

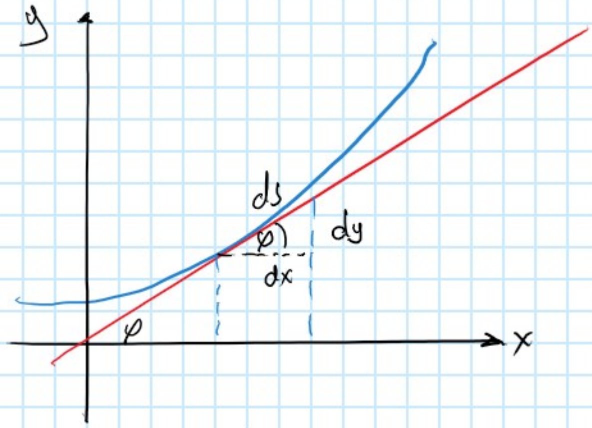
$= \omega V$

Radio de curvatura - Radial y Transveral

Monday, November 5, 2018 4:07 PM

Radio de curvatura : $\vec{\rho}$





$$\tan\varphi = \frac{dy}{dx}$$

$$\frac{d[\tan\varphi]}{dx} = \frac{d\left[\frac{dy}{dx}\right]}{dx}$$

$$\sec^2\varphi \cdot \frac{d\varphi}{dx} = \frac{d^2y}{dx^2}$$

$$\frac{\sec^3\varphi}{\rho} = \frac{d^2y}{dx^2}$$

$$\frac{(1+\tan^2\varphi)^{3/2}}{\rho} = \frac{d^2y}{dx^2}$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx}}$$

$$\rho\, d\varphi = ds \longrightarrow \frac{ds}{dt} = \rho \cdot \frac{d\varphi}{dt}$$

$$\rho = \frac{ds}{d\varphi}$$

$$v = \rho \cdot \frac{d\varphi}{dt}$$

$$v = \rho\, \omega_\rho$$

$$\omega_\rho = \frac{v}{\rho}$$

$$a_N = \frac{v^2}{\rho}$$

$$\rho = \frac{v^2}{a_N}$$

$$\vec{u}_\rho = -\vec{u}_N \longrightarrow \frac{d^2s}{dt^2} = \rho\,\frac{d^2\varphi}{dt^2}$$

$$a_T = \rho \cdot \alpha_\rho$$

$$\rho = \frac{v^2 \cdot v}{v \cdot a \cdot \operatorname{sen}(\theta)}$$

$$\rho = \frac{v^3}{|\vec{v}\times\vec{a}|}$$

$$\cos\varphi = \frac{dx}{ds}$$

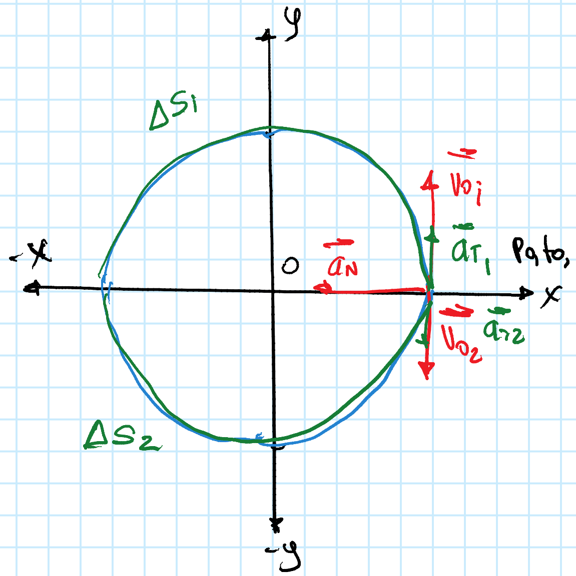
$$\frac{ds}{dx} = \frac{1}{\cos\varphi}$$

$$\frac{\rho \cdot d\varphi}{dx} = \sec\varphi$$

$$\rho \cdot \frac{d\varphi}{dx} = \sec\varphi$$

$$\frac{d\varphi}{dx} = \frac{\sec\varphi}{\rho}$$

9. Dos móviles parten del mismo punto de una circunferencia y tienen la misma rapidez inicial v_0 aunque salen en sentidos opuestos. Uno de los movimientos es acelerado y el otro retardado, pero el módulo de su aceleración y desaceleración respectivamente es el mismo. a) Calcule el valor de la aceleración tangencial a_T sabiendo que el móvil dotado de movimiento retardado en el instante del encuentro lleva velocidad nula. b) Halle la aceleración total de cada uno de los móviles en el momento del encuentro.



$a_T = cte$

$\frac{dv}{dt} = a_T$

$\int_{v_0}^{v} dv = \int_0^t a_T \, dt$

$v - v_0 = a_T \cdot t$

$v = v_0 + a_T \cdot t$

$\cancel{v_2}^{\,0} = -v_0 + a_T \cdot t$

$v_0 = a_T \cdot t$

$t = \frac{v_0}{a_T}$

$t = \frac{v_0}{a_T}$

$v = v_0 + a_T \cdot t$

$v = v_0 + \cancel{a_T}\left(\frac{v_0}{\cancel{a_T}}\right)$

$v = 2v_0$

$a_N = \frac{2v_0}{R}$

$a = \frac{2v_0}{R} + \frac{v_0^2}{\pi R}$

$\Delta S = v_0 t + \frac{1}{2} a t^2$

$\Delta S_1 = v_0 \cdot \frac{v_0}{a_T} + \frac{1}{2} a_T \cdot \left(\frac{v_0}{a_T}\right)^2$

$\Delta S_1 = \frac{v_0^2}{a_T} + \frac{1}{2} \cancel{a_T} \frac{v_0^2}{a_T^{\cancel{2}}}$

$\Delta S_1 = \frac{3}{2} \frac{v_0^2}{a_T}$

$\Delta S_2 = -v_0\left(\frac{v_0}{a_T}\right) + \frac{1}{2} a_T \left(\frac{v_0}{a_T}\right)^2$

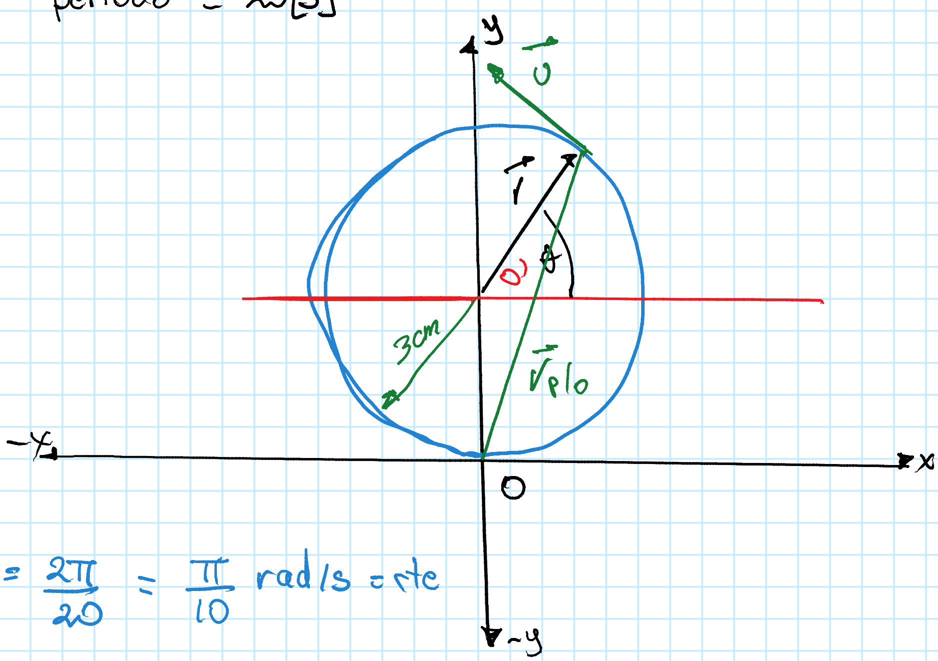
$\Delta S_2 = -\frac{1}{2} \frac{v_0^2}{a_T}$

$|\Delta S_1| + |\Delta S_2| = 2\pi R$

$\frac{3}{2}\frac{v_0^2}{a_T} + \frac{1}{2}\frac{v_0^2}{a_T} = 2\pi R$

$\cancel{2} \frac{v_0^2}{a_T} = \cancel{2}\pi R$

periodo = 20[s]



$$\omega = \frac{2\pi}{20} = \frac{\pi}{10} \text{ rad/s} = \text{cte}$$

$$\theta = \theta_0 + \omega t$$

$$\theta = \frac{3\pi}{2} + \frac{\pi}{10} t$$

$$\vec{r} = R \cos\theta \, \vec{i} + R \operatorname{sen}\theta \, \vec{j}$$

$$\vec{r} = R \cos\left(\frac{3\pi}{2} + \frac{\pi}{10} t\right)\vec{i} + R \operatorname{sen}\left(\frac{3\pi}{2} + \frac{\pi}{10} t\right)\vec{j}$$

$$\vec{r}_{P/O} = 3\vec{i} + \left[R \cos\left(\frac{3\pi}{2} + \frac{\pi}{10} t\right)\vec{i} + R \operatorname{Sen}\left(\frac{3\pi}{2} + \frac{\pi}{10} t\right)\vec{j}\right] \quad \text{rad}$$

$$\vec{V} = -R\omega . \operatorname{sen}(\theta)\,\vec{i} + R\omega \cos(\theta)\,\vec{j}$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

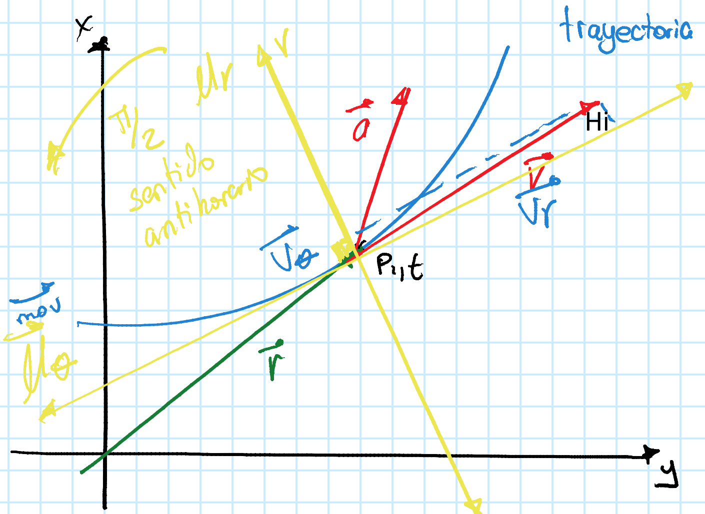
$$= \omega . \vec{k} \times \left[R . \cos(\theta)\,\vec{i} + R \operatorname{sen}(\theta)\,\vec{j}\right]$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$
$$= \omega.\vec{k} \times [R.\cos(\theta)\vec{i} + R\,\text{sen}(\theta)\vec{j}]$$
$$= -\omega R\,\text{sen}\,\theta\,\vec{i} + \omega R\cos\theta\,\vec{j}$$

Coordenadas polares. Solo se definen en el plano

Componentes: Radial - Transversal

$\vec{r} = (r, \theta)$



Eje radial: r, $\vec{u_r}$

Eje transversal: θ, $\vec{u_\theta}$

$$\vec{r} = \vec{r_r} + \cancel{\vec{r_\theta}}$$
$$\vec{r} = r\,\vec{u_r}$$
$$\vec{v_r} = \frac{\vec{v}\cdot\vec{r}}{r^2}\,.\,\vec{r}\,, \quad \vec{v} = \vec{v_r} + \vec{v_\theta}$$

$$\vec{a_r} = \frac{\vec{a}\cdot\vec{r}}{r^2}\,.\,\vec{r}\,, \quad \vec{a} = \vec{a_r} + \vec{a_\theta}$$

$$\vec{u_r} = \cos\theta\,\vec{i} + \text{sen}\,\theta\,\vec{j}$$
$$\vec{u_\theta} = -\text{sen}\,\theta\,\vec{i} + \cos\theta\,\vec{j}$$

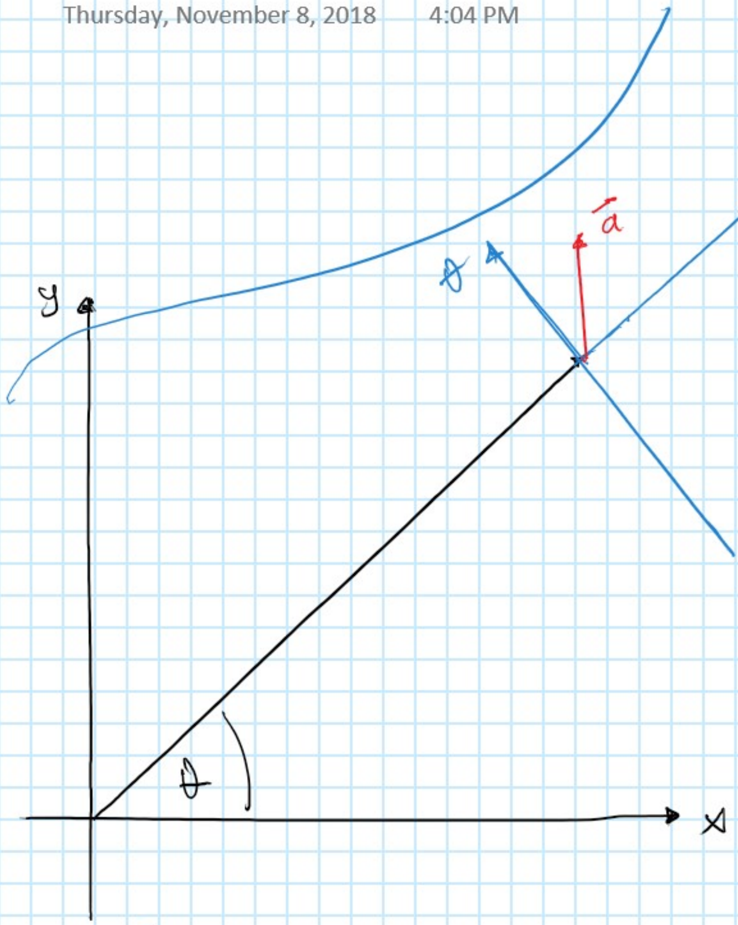
$$\frac{d\vec{u_r}}{dt} = -\text{sen}\,\theta.\frac{d\theta}{dt}\vec{i} + \cos\theta.\frac{d\theta}{dt}\vec{j}$$
$$= [-\text{sen}\,\theta\,\vec{i} + \cos\theta\,\vec{j}]\frac{d\theta}{dt}$$
$$= \dot{\theta}[-\text{sen}\,\theta\,\vec{i} + \cos\theta\,\vec{j}]$$
$$= \dot{\theta}\,\vec{u_\theta}$$

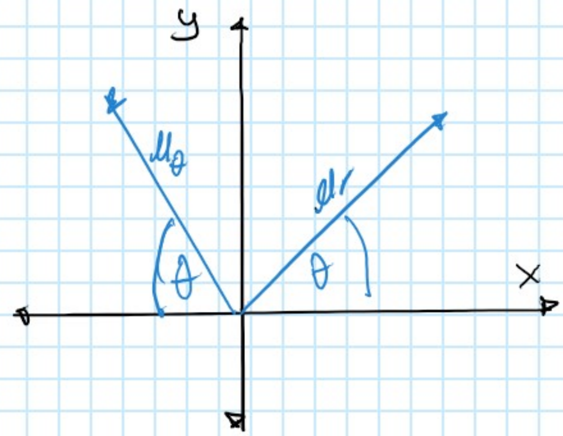
$$\frac{d\vec{u_\theta}}{dt} = -\cos\theta.\frac{d\theta}{dt}\vec{i} - \text{sen}\,\theta.\frac{d\theta}{dt}\vec{j}$$
$$= -\dot{\theta}[\cos\theta\,\vec{i} + \text{sen}\,\theta\,\vec{j}]$$

$$\frac{d\vec{u_\theta}}{dt} = -\dot{\theta}[\cos\theta\,\vec{i} + \text{sen}\,\theta\,\vec{j}]$$
$$= -\dot{\theta}\,\vec{u_r}$$

Radial Transveral y Dinámica

Thursday, November 8, 2018 4:04 PM





$$\frac{d\vec{u_r}}{dt} = \dot\theta\,\vec{u_\theta}$$

$$\frac{d\vec{u_\theta}}{dt} = -\theta\,\vec{u_r}$$

$\vec r\ (r, \theta)$

$u_r = (\cos\theta\,\vec i + \operatorname{sen}\theta\,\vec j)$

$u_T = (-\operatorname{sen}\theta\,\vec i + \cos\theta\,\vec j)$

$\vec V = r.\vec{u_r}$

$$\vec V = \frac{d\vec r}{dt} = \frac{d(r\vec{u_r})}{dt} = \frac{dr}{dt}\vec{u_r} + \dot\theta\vec{u_\theta}\,r$$

$$= \frac{dr}{dt}.\vec{u_r} + \frac{d\vec{u_r}}{dt}.r$$

$\vec V = \vec{V_r} + \vec{V_\theta}$

$\vec{V_r} = \frac{dr}{dt}.\vec{u_r}$ $\qquad \vec{V_\theta} = r.\dot\theta\,\vec{u_\theta}$

$\vec{V_r} = \dot r\,\vec{u_r}$ $\qquad V_\theta = r.\dot\theta$

$V_r = \dot r$ $\qquad \downarrow$

ω_r

$\vec V = \dot r\,\vec{u_r} + r.\dot\theta\,\vec{u_\theta}$

$$\frac{d\vec v}{dt} = \vec a = (\ddot r\,\vec{u_r} + \dot r\dot\theta\,\vec{u_\theta}) + \dot r\dot\theta\,\vec{u_\theta} + r\ddot\theta\,\vec{u_\theta} + r\dot\theta\,(-\dot\theta\,\vec{u_r})$$

$\vec{a_r} = (\ddot r - r\dot\theta^2)\vec{u_r}$ $\qquad \vec{a_T} = (2\dot r\dot\theta + r\ddot\theta)\,\vec{u_\theta}$

$a_r = \ddot r - r\dot\theta^2$ $\qquad a_T = 2\dot r\dot\theta + r\ddot\theta$

Trayectoria una circunferencia coincidente en el eje de coordenadas

Trayectoria una circunferencia coincidente en el eje de coordenadas

13. En un instante dado, una partícula tiene la siguiente posición, velocidad y aceleración relativos a un sistema de coordenadas $x-y$, $x=4\text{m}$, $y=2\text{m}$, $\dot{x}=2\sqrt{3}\text{m/s}$, $\dot{y}=-2\text{m/s}$, $\ddot{x}=-5\text{m}/s^2$, $\ddot{y}=5\text{m}/s^2$. Determine θ, $\dot{\theta}$, $\ddot{\theta}$, r, $\dot{r}$, $\ddot{r}$.

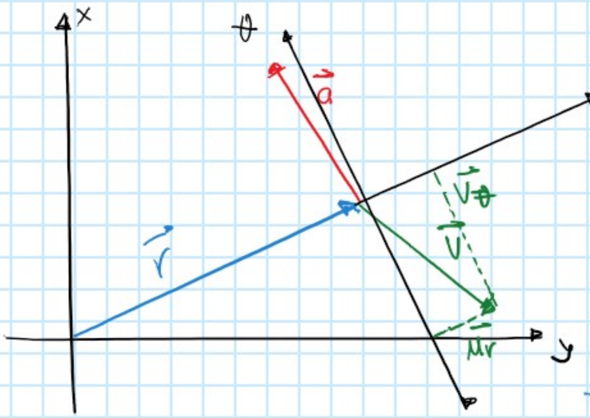
$\vec{r} = 4\vec{i} + 2\vec{j}$ [m]

$\vec{v} = 2\sqrt{3}\,\vec{i} - 2\vec{j}$ [m/s]

$\vec{a} = -5\vec{i} + 5\vec{j}$ [m/s²]

$\vec{r} = r \cdot \vec{u}_r$

$2\sqrt{5}\,\vec{u}_r$



$\vec{v}_r = \vec{v}_{\vec{r}} =$

$v_r = \frac{\vec{v}\cdot\vec{r}}{r}$

$v_r = \frac{(4)(2\sqrt{3}) + (2)(-2)}{\sqrt{20}}$

$v_r = 2.2 = \dot{r}$

$\tan\theta = \frac{2}{4}$

$\theta = 0.4636$ rad

$r = \sqrt{20}$

$a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta}$

$6.71 = 2\cdot(2.2)(-0.74) + \sqrt{20}\cdot\ddot{\theta}$

$\ddot{\theta} = 2.22$ rad/s²

$v^2 = v_r^2 + v_\theta^2$

$4^2 = (2.2)^2 + v_\theta^2$

$v_\theta = -3.34$ m/s $= r\cdot\dot{\theta}$

$\dot{\theta} = \frac{v_\theta}{r} = \frac{-3.34}{\sqrt{20}} = -0.7468$

$a_r = \frac{\vec{a}\cdot\vec{r}}{r} = \frac{-10}{2\sqrt{5}} = -2.2360$

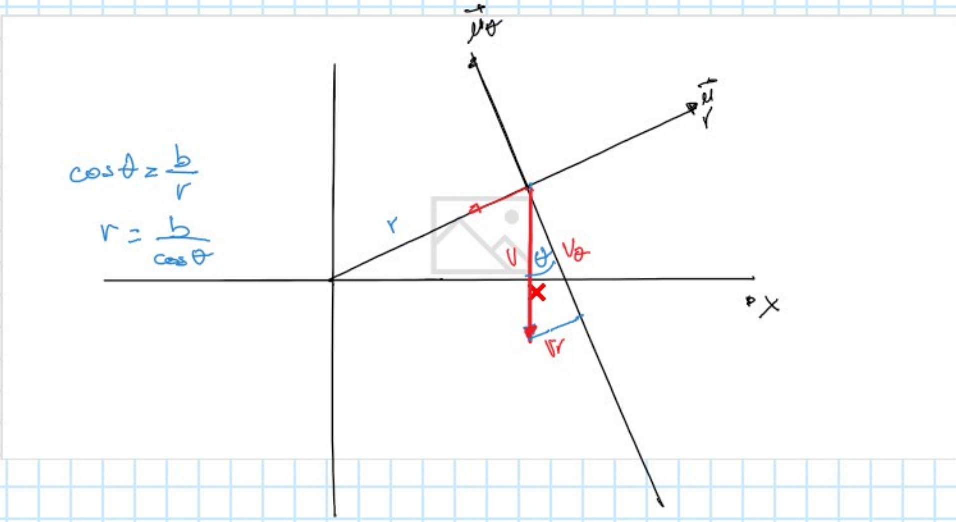
$a_\theta = \sqrt{50 - (-2.2360)^2} = 6.7102$ m/s

$a_r = \ddot{r} - r\dot{\theta}^2$

$-2.2360 = \ddot{r} - \sqrt{20}\,(-0.7468)^2$

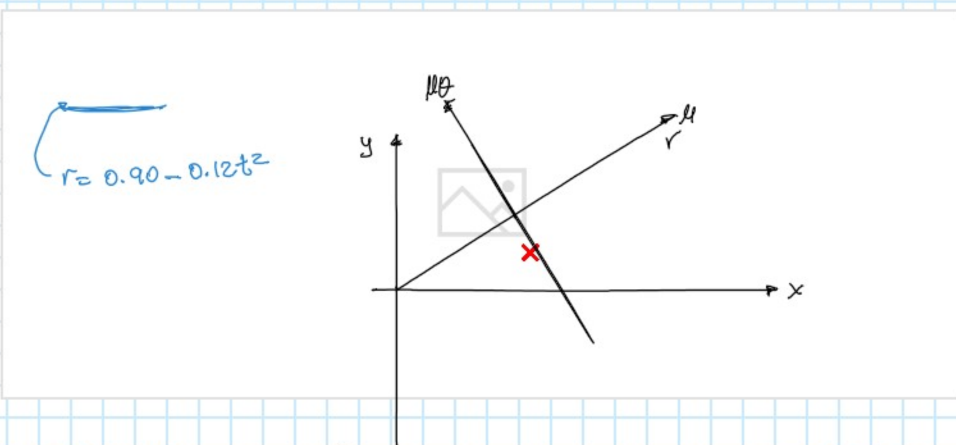
$$-2.2360 = \ddot{r} - \sqrt{20}\,(-0.7468)^2$$

$$\ddot{r} = 0.2581 \ m/s^2$$



$$\cos\theta = \frac{V_\theta}{V} \qquad V = \frac{V_\theta}{\cos\theta}$$

$$V = \frac{r\,.\,\dot{\theta}}{\cos\theta} = \frac{b\ \dot{\theta}}{\cos^2\theta}$$



$$\theta = 0.15t^2$$

$$\frac{\pi}{6}\ ,\quad t = 1.86\ [s]$$

$\theta = 0.15t^2$ [rad]	0.5235	$r = 0.90 - 0.12t^2$	[m]	0.4811 [m]
$\dot{\theta} = 0.3t$	[rad/s]	0.56 rad/s	$\dot{r} = -0.24t$.	[m/s]	-0.44 [m/s]
$\ddot{\theta} = 0.3$	[rad/s²]	0.3 rad/s	$\ddot{r} = -0.24$	[m/s²]	-0.24 [m/s²]

$\theta = \frac{\pi}{6}$ rad

$$V_r = \dot{r} = -0.44 \ m/s \qquad a_r = \ddot{r} - r\dot{\theta}^2 = -0.39 \ m/s^2$$

$$V_\theta = r\,.\,\dot{\theta} = 0.2688 \ m/s \qquad a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta} = -0.316 \ m/s^2$$

Dinámica de la partícula

Objetivo:

Dinámica

Relacionar con las causas que generan el movimiento interacciones con los cuerpos que lo rodean

Interacciones

↳ acción recíproca

4 fundamentales.

- Gravitacional
- Electromagnética ↳ de contacto
- Fuerte
- Débil

Fuerza $\vec{F}$

- magnitud
- dirección

grado de interacción entre los cuerpos.

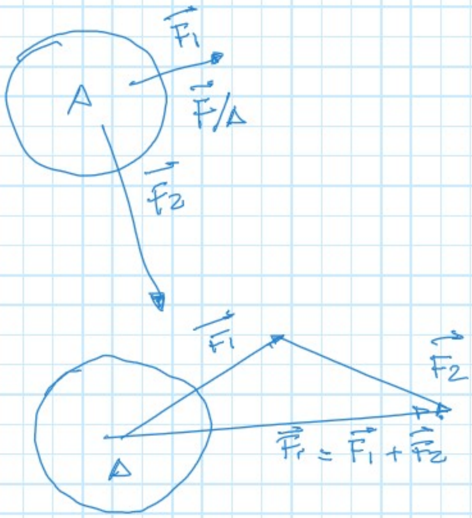
Leyes de Newton

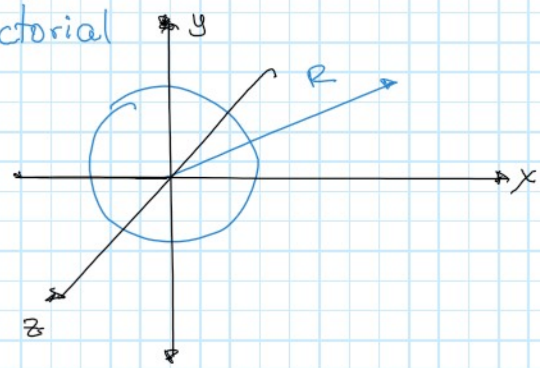
Monday, November 12, 2018 4:06 PM

Fuerza $\vec{F}$

grado de interacción entre los cuerpos

cantidad de tipo vectorial





Fueza Externa Neta (Fuerza resultante o neto) $\sum \vec{F}$

↳ Suma vectorial

Representación de una fuerza.

↳ Estrictamente no es una fuerza

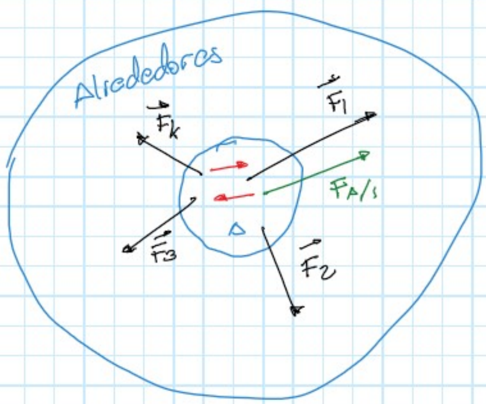
Fuerzas externas

$\vec{F_1}, \vec{F_2}, \vec{F_3}, \ldots, \vec{F_k}$

↳ Realizan los agentes externos sobre el cuerpo

Fuerzas internas

Fuerzas que realiza el cuerpo



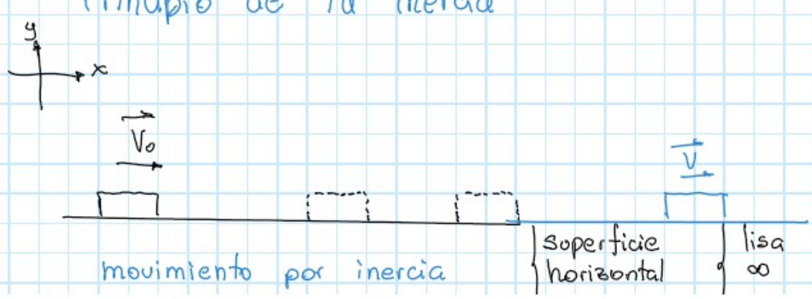
$$\vec{F_1} + \vec{F_2} + \vec{F_3} + \ldots + \vec{F_k} = \vec{F_N}$$

$$\sum_{n=1}^{k} \vec{F_i} = \vec{F_N}$$

Leyes de Newton

Primera ley

Principio de la inercia



movimiento por inercia

$\vec{F_N} = 0$

superficie horizontal rugosa. | lisa ∞

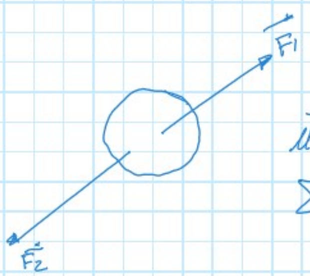
Inercia

↳ tendencia a mantener su estado de reposo o movimiento

① Todo cuerpo conserva su estado de reposo o de movimiento rectilíneo uniforme mientras no exista una fuerza neta que lo obligue a cambiar

Equilibrio

$$\sum \vec{F} = \vec{0} \Leftrightarrow \vec{v} = \vec{v}_{cte} \begin{cases} = \vec{0} & \text{Reposo} \\ \neq \vec{0} & \text{M.R.U.} \end{cases}$$



$\vec{u}_{F_1} = -\vec{u}_{F_2}$

$\sum \vec{F} = \vec{F_1} + \vec{F_2} = 0$

$\vec{v} = \vec{cte}$ $\vec{a}$

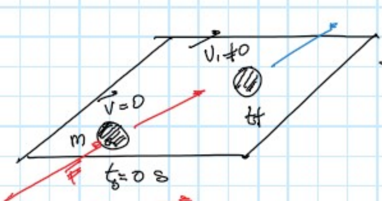
- Sistemas inerciales de referencia ⇒ Idealización (no existe ninguno)
  - ↳ se cumple la primera ley
  - ↳ se encuentra en reposo o con velocidad constante
- Sistemas no inerciales.
  - ↳ Poseen aceleración respecto de un sistema inercial

Cuerpo n gran masa y tamaño

↳ sistema inercial

Segunda ley: Principio de la fuerza

$$\sum \vec{F} = \vec{F} \neq \vec{0}$$



superficie horizontal lisa

$\vec{a} \propto \sum \vec{F_N}$ | $a \propto \sum F$

$\vec{u}_a = \vec{u}_{\Sigma F}$

$\vec{F} = \vec{cte} =$ horizontal

} Inercia

$t_0 = 0\,s$

$\vec{F} = cte$ = horizontal

La fuerza neta diferente de cero que actúe sobre un cuerpo, imprime en él una aceleración directamente proporcional a la fuerza neta e inversamente proporcional a la inercia

Inercia

Masa : m , [kg]

Cantidad arbitraria

$\alpha = \frac{1}{m}$ $\quad \sum_i \vec{F}$

$\vec{a} = \frac{\sum \vec{F}}{m}$ $\quad \sum \vec{F} \propto m \cdot \vec{a}$

$\sum_i \vec{F} \propto m \cdot \vec{a}$

$\sum_i \vec{F} = \vec{k}\, m \cdot \vec{a} \leftarrow$ (instantanea)

$MLT^{-2} = MLT^{2}$ $\quad \sum_i \vec{F} = m \cdot \vec{a}$

kg / m SI s

$N = kg\, m/s^2$

componentes rectangulares

1. $F_x \vec{i}$
2. $F_j \vec{j}$
3. $F_k (\vec{x})$

$\sum_i F_x \vec{i} + \sum_i \vec{F}_y + \sum_i \vec{F}_z + \vec{R} = m(a_x \vec{i} + a_y \vec{j} + a_k \vec{a_k})$

$= m\vec{a_x} + m\vec{a_y} + m\vec{a_z}$

$\sum_i F_x = m \cdot a_x$, $\sum_i F_y = m \cdot a_y$, $\sum_i F_z = m \cdot a_z$

Componentes, tangencial y normal

$\sum_i \vec{F_N} = m(\vec{a})$

$\sum \vec{F_T} + \underset{\text{Fuego}}{\sum_i F_n} = m(a_t + a_N)$

$\sum \vec{F_y} + \sum_i \vec{F_x} = m\vec{a_t} + m\vec{a_N}$

Fuerza centripeta, aceleración normal

Componentes Radial y Transversal

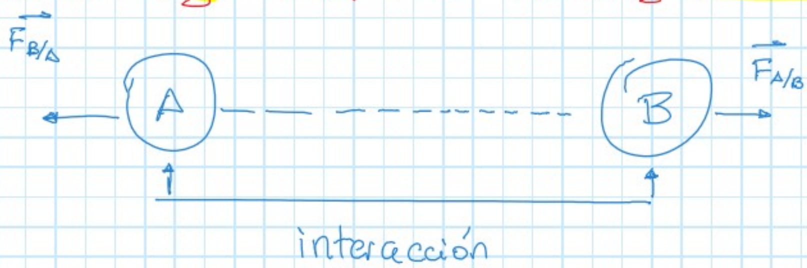
$\sum_i \vec{F} = m(\vec{a_r} + \vec{a_\theta})$

$\sum_i \vec{F_r} = m\,\vec{a_r}$

$\sum_i \vec{F_\theta} = m \cdot \vec{a_\theta}$

Tercera ley: Principio de acción y reacción

Tercera ley: Principio de acción y reacción



Si un cuerpo A, ejerce una fuerza sobre un cuerpo B. El cuerpo B también ejerce una fuerza sobre el cuerpo A de igual magnitud pero dirección contraria pero en la misma linea de acción

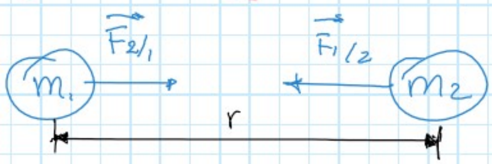
$$|\vec{F}_{A/B}| = |\vec{F}_{B/A}|$$

$$\vec{u}_{F_{A/B}} = -\vec{u}_{F_{B/A}}$$

Denominación arbitraria

Caracter de simultaneedad

Ley de la gravitación universal



fuerza central

$$\vec{F}_G = G\frac{m_1 m_2}{r^2}$$

$G = 6.67 \times 10^{-11}$

D.C.L

↳ Diagrama de cuerpo libre

Representación gráfica de un cuerpo y de las fuerzas externas que actuan sobre el.

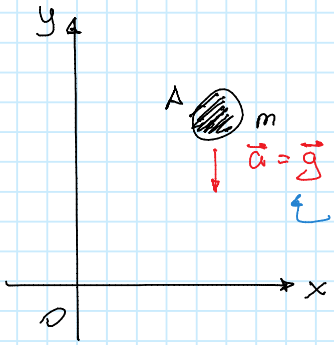
① Aislar al cuerpo.

② Identifico con que cuerpos interactua el cuerpo que se está analizando

③ Dibujar las fuerzas que resultan de esas interacciones

Peso: $\vec{P}$, $\vec{w}$

Fuerza que resulta de la acción gravitacional que resulta de la acción gravitacional que ejerce la masa de la Tierra sobre el cuerpo.



interacción gravitacional hacia el centro de la tierra.

D.C.L : A

$$\sum \vec{F} = m.\vec{a}$$

$$\vec{a} = \vec{g}$$

$$\vec{P} = m.\vec{g}$$

magnitud $P = m.g$

dirección $u_{\vec{P}} = u_{\vec{g}}$

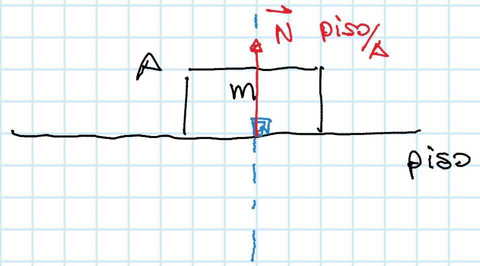
Fuerzas

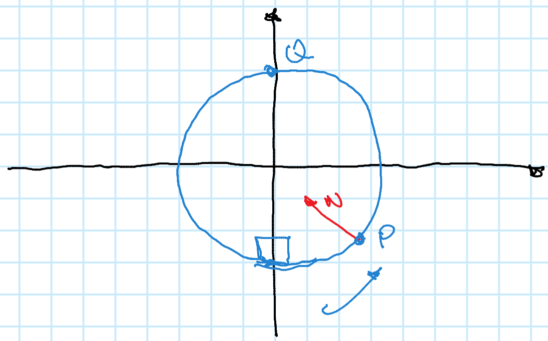
Thursday, November 15, 2018 4:10 PM

Normal de Contacto : $\vec{N}$

balanza → normal de contacto

normal y peso actuan sobre el mismo cuerpo





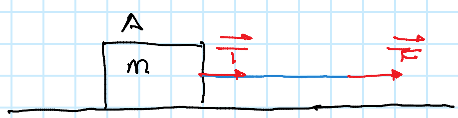
Tensión : $\vec{T}$

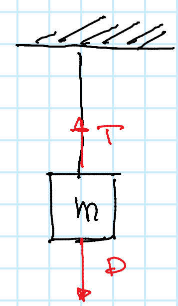
Fuerza que se aplica a un cuerpo mediante una cuerda

cuerda ⇔ ideal

- flexible
- inextensible
- masa despreciable

cuerda = línea de acción
la fuerza se transmite de extremo a extremo





Fuerza de rozamiento $\vec{fr}$

Seco o de Coulomb

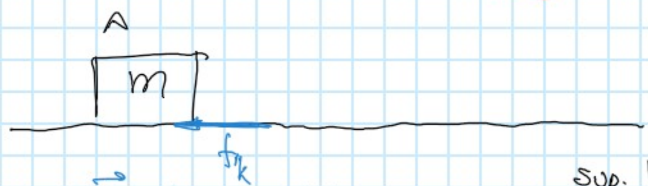
Tendencia al movimiento
Movimiento

relativas entre las superficies en contacto ⇐ dirección contraria

tangente a la superficie de contacto

$t_3, F_3 > F_2$

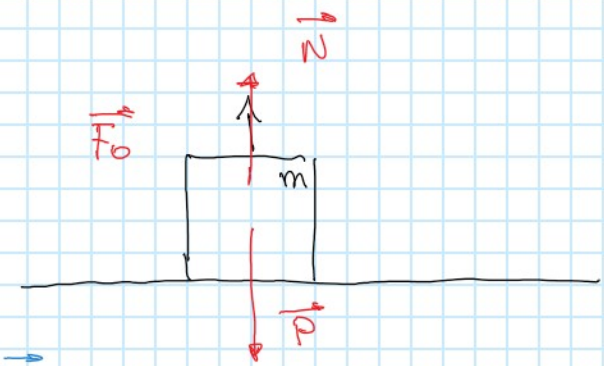
μ →



$\vec{F}$ = horizontal

sup. horizontal rugosa

$t_0 = 0, \quad \vec{F_0} = 0$



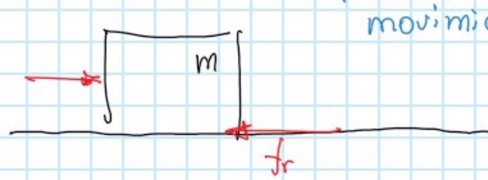
$\sum_i \vec{F} = 0$

$\sum_i F_x = 0 \qquad \sum_i F_x = 0$

$F_0 = 0 \qquad N + (-P) = 0$

$t_1, F_1 \neq 0$

tendencia al movimiento



$\sum_i F_x = 0$

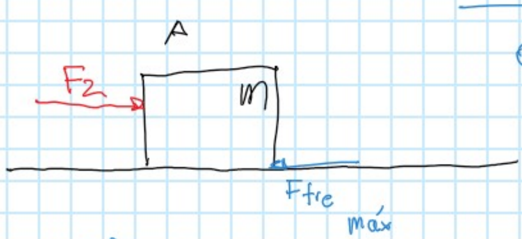
$\sum_i F_x = 0$

$F_1 + (-f_{r_s}) = 0$

$t_2, F_2 > F_1$

mov inminente →

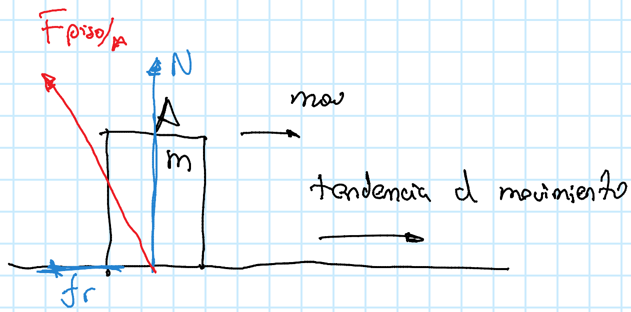
$0 < f_{r_s} < f_{r_{s\,máx}}$

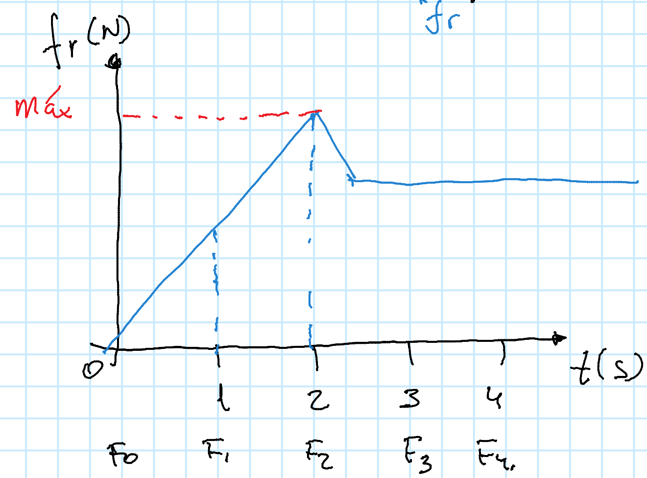


$f_r \propto \vec{N}$ $\quad \mu$ = coeficiente de rozamiento

$fr_k = \vec{N}\,\mu_k$

$fr_s = \vec{N}\,\mu_s$





Resistencia $\vec{R}$

$\vec{R}$

- magnitud $R = f(v) \longrightarrow R = kv$
- dirección contraria al movimiento

velocidad máxima / terminal, cuerpo que se mueve bajo la acción de una resistencia.

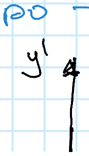
↳ cuando la aceleración se hace 0

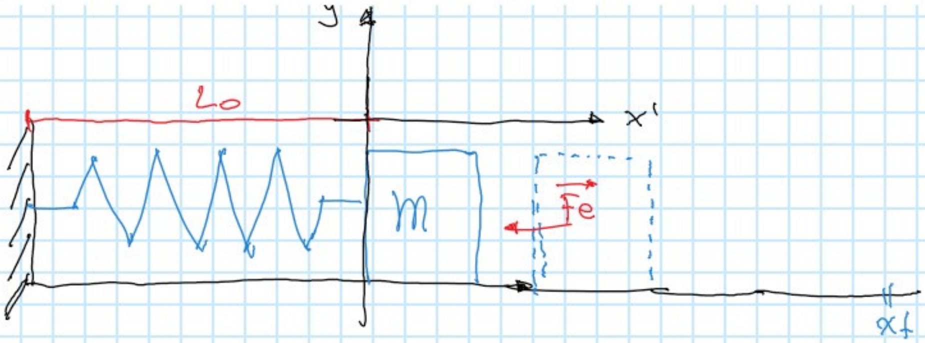
Fuerza elástica: $\vec{Fe}$

Sistema deformable

{ Posiciones relativas de las partículas del sistema cambian

masa cuerpo — resorte

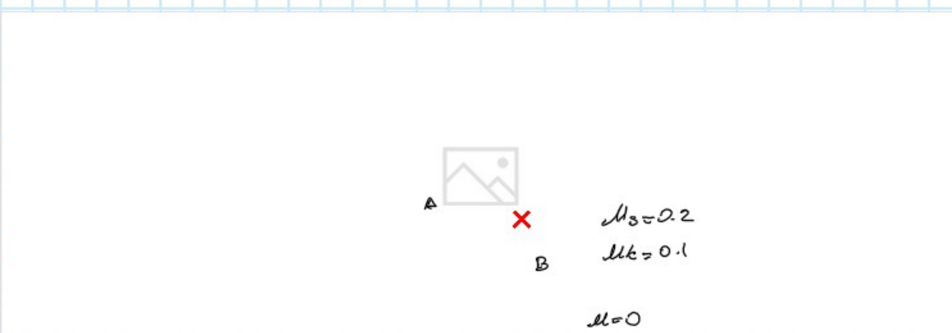




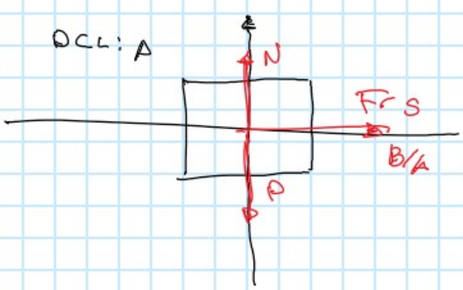
Ley de Hook

$$Fe = -kx$$

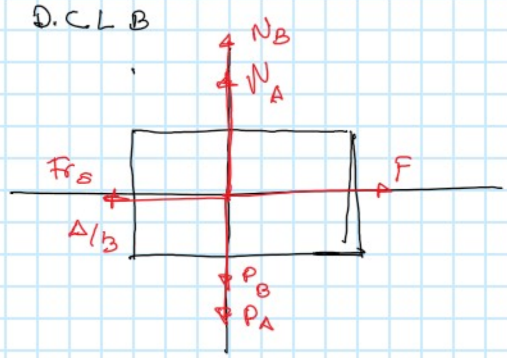
k : constante elástica del resorte
x : deformación (posición)



DCL: A



D.C.L B



$$\sum F_x = m_A a_{A_x}$$

$$fr_{máx} = m_A a_x$$

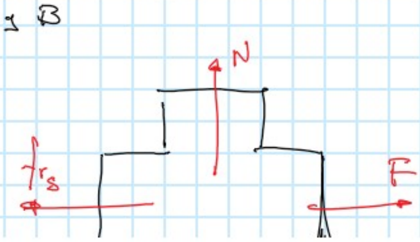
$$\mu_s . N = m_A a_x$$

$$\mu_s \cancel{m_A} . g = \cancel{m_A} . a_x$$

$$\mu_s . g = a_x$$

$$a_{máx} = 1,96 \ m/s^2$$

DCL : A y B

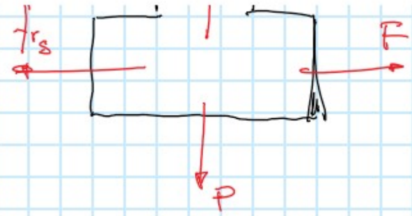


$\mu s . g = a_x$

$a_{máx} = 1,96 \ m/s^2$

$F_{máx} = 1,96(8)$

$= 15,62 \ [N]$



$\sum \vec{F_x} = m . a_x$

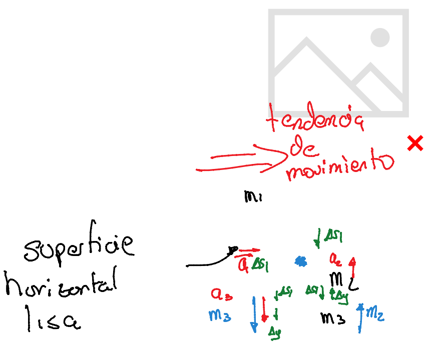
$\vec{F} = 8 . a_x$

$\sum F_y = 0$

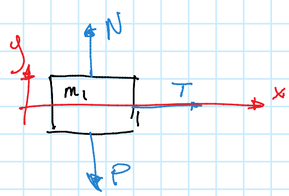
Dinámica

Monday, November 19, 2018 4:25 PM

$$\frac{a_1}{4 m_2 \; m_3} = P$$



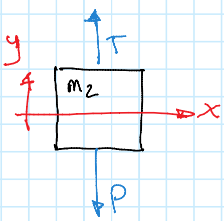
D.C.L.(1)



$\sum_i F_x = m_1 a_1$

$T_1 = m_1 . a_1$

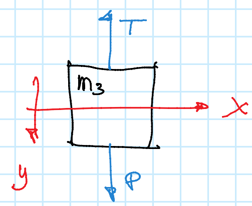
D.C.L (2).



$\sum_i F_y = m_2 a_2$

$T + (-P_2) = m_2 a_2$

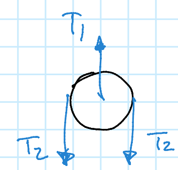
D.C.L. 3.



$\sum_i F_y = m_3 a_3$

$P_3 + (-T) = m_3 a_3$

D.C.L (4)



$\sum_i F_y = \cancel{m_{Pm}}^{0} \cancel{a_{Pm}}^{0}$

$2T + (-T_1) = 0$

$2T = T_1$

$\Delta S_3 = \Delta S_1 + \cancel{\Delta y}$

$\Delta S_1 = \cancel{\Delta y} - \Delta S_1$

$\Delta S_3 - \Delta S_2 = 2 \Delta S_1$

$V_3 - V_2 = 2 V_1$

$a_3 - a_2 = 2 a_1$

$$\begin{cases} T_1 = m_1 a_1 \\ T + (-P_2) = m_2 a_2 \\ P_3 + (T) = m_3 a_3 \\ 2T = T_1 \\ a_3 - a_2 = 2a_1 \end{cases}$$

$a_1 = 2T / m_1$

$a_2 = (T - P_2) / m_2 =$

$a_3 = (P_3 - T)\, m_3 =$

$a_1 = T_1 / m_1$

$a_2 = (T - m_2 g) / m_2$

$a_3 = (m_3 g - T)\, m_3$

$$a_3 - a_2 = g - \frac{T}{m_3} - \frac{T}{m_2} + g$$

$$2 a_1 = 2g - \frac{(m_2 - m_3)}{m_3 m_2} T$$

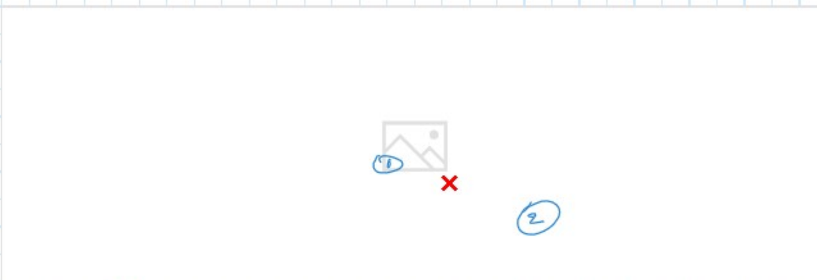
$$2 a_1 + \frac{m_2 + m_3}{m_2 m_3} \; \frac{m_1}{2} \cdot a_1 = 2g$$

$$a_1\left[2+\frac{m_1 m_2+m_1 m_3}{2m_2 m_3}\right]=2g$$

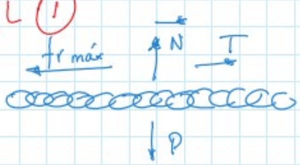
$$a_1\left[\frac{4m_2 m_3+m_1 m_2+m_1 m_3}{2m_2 m_3}\right]=2g$$

$$a_1=\frac{4m_2\,m_3\;g}{4\,m_2\,m_3}$$

Reemplazo de P.



D.C.L (1)



$\Sigma F_x=0$

$T+(-f_{r\,máx})=0$

$T-\mu m_1 g=0$

$T=\mu m_1 g$

$\Sigma F_y=0$

$N=mg$

D.C.L. (2)

$\Sigma F_y=0$

$P_2+(-T)=0$

$P_2=T$

$m_2 g=T$

$m_2\cancel{g}=\mu.m_1\cancel{g}$

$m_2=\mu.m_1$

$\cancel{\lambda}b=\mu(L-b)\cancel{\lambda}$

$b=\mu(L-b)$

$b=\mu L-\mu b.$

$b(1+\mu)=\mu.L$

$\boxed{b=\frac{\mu L}{1+\mu}}$

densidad lineal

$\lambda=\frac{m}{L}$

$m=\lambda L$

Parte b)

$\Sigma F_x=m_1.a_1$

$T-f_{rc}=m_1 a_1$

$T-\mu m_1 g=m_1 a_1$

$T=m_1 a_1+\mu m_1 g$

$\Sigma F_y=m_2 a_2$

$P_2-T=m_2 a_2$

$m_2 g-T=m_2 a_2$

$T=m_2 g-m_2 a_2$

$m_2 g-\mu m_1 g=(m_1+m_2)a$

$m_2 g-\mu m_1 g=m\,a$

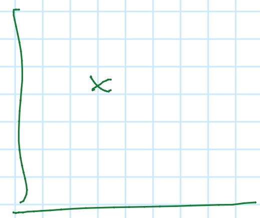
$\cancel{\lambda}yg-\mu\cancel{\lambda}(L-y)=\cancel{\lambda}.L\,a$

$yg-\mu(L-y)=L\,a$

$$a = \frac{g}{L}\left[y - \mu L + \mu y\right]$$

$$a = \frac{g}{L}\left[y(1+\mu) - \mu L\right]$$

$$\frac{dy}{dy}\cdot\frac{dv}{dt} = a = \frac{g}{L}\left[y(1+\mu) - \mu L\right]$$



$$v\cdot\frac{dv}{dy} = \frac{g}{L}\left[y(1+\mu) - \mu L\right]$$

$$\int_0^v v\cdot dv = \int_{0.5}^{3}\frac{g}{L}\left[y(1+\mu) - \mu L\right] dy$$

$$= \frac{g}{L}\int_{0.5}^{3}\left[y + \mu y - \mu L\right] dy$$

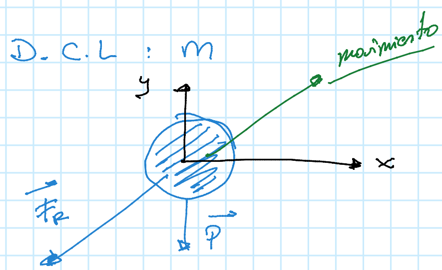
$$= \frac{g}{L}\left[\int_{0.5}^{3} y + \mu y - \int_{0.5}^{3}\mu L\right.$$

$$= \frac{g}{L}\left[\left.\frac{y^2}{2} + \frac{\mu y^2}{2}\right|_{0.5}^{3} - \mu L\right]$$

$$= \frac{g}{L}\left[\frac{21}{4} - \frac{3}{5}\right]$$

$$= 15.19 \qquad \boxed{\sqrt{6.06}}$$

D.C.L : m



$$\Sigma F_x = m.\vec{a}$$

$$-k V_x \vec{i} = m.a$$

$$\frac{dv}{dt}.\ a = \frac{-k V_x}{m}\vec{i}$$

$$\int_{v_0}^{v}\frac{dv}{V_x} = \int_0^t \frac{-k}{m}\, dt$$

$$\vec{F_r} = -k\vec{v}$$

$$\vec{F_r} = -k\vec{v}$$
$$= -k(v_x\vec{i} + v_y\vec{j})$$
$$= (-k v_x\vec{i}) + (-k v_y\vec{j})$$

$$\int_{v_0} \frac{dv}{v_x} = \int_0 \frac{-k}{m}\, dt$$

$$-\frac{k}{m}t = \ln(v)\Big|_{v_0}^{v_x}$$

$$-\frac{k}{m}t = \ln\left(\frac{v_x}{v_0}\right)$$

$$e^{-\frac{k}{m}t} = \frac{v_x}{v_0}$$

$$\int_0^x v_x = \int_0^t v_0 \cdot e^{-\frac{k}{m}\cdot t}$$

$$x = v_0 \int_0^t e^{-\frac{k}{m}\cdot t}$$

$$x = v_{0x} \int_0^t e^{-\frac{k}{m}t}$$

$$x = v_{0x}\left(-\frac{m}{k}\right) \cdot e^{-\frac{k}{m}\cdot t}\Big|_0^t$$

$$x = v_{0x}\left(-\frac{m}{k}\right)\left[e^{-\frac{k}{m}t} - 1\right]$$

$$x = v_{0x}\left(\frac{m}{k}\right)\left[1 - e^{-\frac{k}{m}t}\right]$$

e^{u}

$$\sum_i F_y = m a_y$$
$$-mg - kv = m a_y$$

$$\frac{dv_y}{dt} = a_y = -\left[g + \frac{k}{m} v_y\right]$$

$$-\left(g + \frac{k}{m} v_y\right) = \frac{dv_y}{dt} \cdot \frac{dy}{dy}$$

$$-\left(g + \frac{k}{m} v_y\right) = \frac{v_y\, dv_y}{dy}$$

$$-dy = \frac{v_y\, dv_y}{-g + \frac{k}{m} v_y}$$

$- y_{máx}$

$v_{y} = 0$

$$u = g + \frac{k}{m} v_y \Rightarrow \frac{m}{k}(u - g) = v_y$$

$$du = \frac{k}{m}\, dv_y$$

$$\frac{m}{k}\, du = dv_y$$

$$\int \frac{\frac{m}{k}(u-g)\frac{m}{k}\,du}{u}$$

$$\frac{m^2}{k^2}\int\left[1-\frac{g}{u}\right]du$$

$$\frac{m^2}{k^2}\left[u-g\ln(u)\right]$$

$$-y_{máx}=\frac{m^2}{k^2}\left[\left(g+\frac{k}{m}v_y\right)-g\ln\left(g+\frac{k}{m}v_y\right)\right]\Big|_{v_{y0}}^{0}$$

$$-y_{máx}=\frac{m^2}{k^2}\left[\left(g-g-\frac{k}{m}v_{y0}\right)-g\ln\left(\frac{g}{g+\frac{k}{m}v_{y0}}\right)\right]$$

$$y_{máx}=\frac{m^2}{k^2}\left[\frac{k}{m}v_{y0}+g\ln\left(\frac{g}{g+\frac{k}{m}v_0}\right)\right]$$

$$a_N=g\cos(\theta_0)$$

$$a_T=g^2-a_N^2$$

$$a=-\frac{k}{m}V_{0x}\vec{i}-\left(g+\frac{k}{m}v_y\right)\vec{j}$$

$$a=\left(-\frac{k}{m}v_{0x}\vec{i}\right)-\left(g+\frac{k}{m}\vec{v_y}\right)\vec{j}$$

Momento Lineal e Impulso

Thursday, November 22, 2018 4:05 PM

Impulso lineal y cantidad de movimiento lineal

da ley $\sum \vec{F} = m \cdot \vec{a}$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\sum \vec{F} = m \cdot \frac{d\vec{v}}{dt}$$

Daniel lara

$$\sum \vec{F} = \frac{d(m.\vec{v})}{dt}$$

cantidad de moviento lineal: $\vec{p}$

↳ cuantifica el movimiento

$$\vec{p} = m.\vec{v}. \quad \frac{kg\,m}{s}$$

instantanea

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

fuerza motriz

$$\sum \vec{F} = \frac{d(m\vec{v})}{dt}$$

$$\sum \vec{F} = \frac{dm}{dt}v + \frac{d\vec{v}}{dt}m$$

$$\sum \vec{F} = \dot{m}\frac{d\vec{r}}{dt} + m.\frac{d^2 r}{dt^2}$$

$$\dots \frac{\dots}{dt} + m \frac{\dots}{dt^2}$$

$$\sum \vec F = \frac{d\vec p}{dt}$$

Impulso lineal $\vec I$

- intensidad
- duración

$$\int_0^t \sum \vec F \, dt = \int_{p_0}^{p_f} d\vec p$$

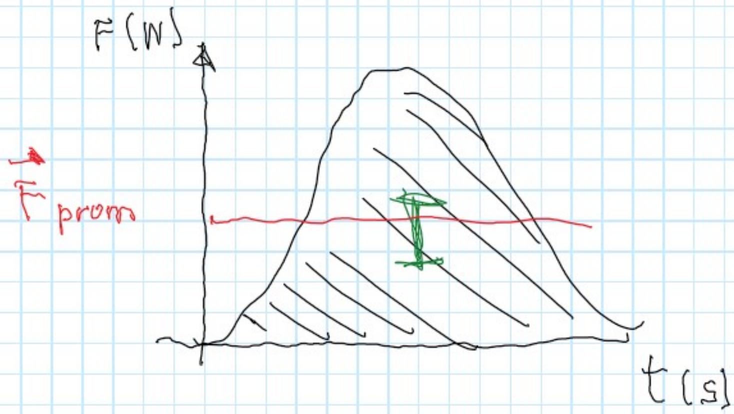
$$I = \int_{t_0}^{t_f} \vec F \, dt$$

$\vec F$ → función vectorial del tiempo

$$\vec I_{NETO} = \vec p \Big|_{p_0}^{p_f}$$

$$= \vec p_f - \vec p_0$$

$$\vec I_{NETO} = \Delta \vec P$$

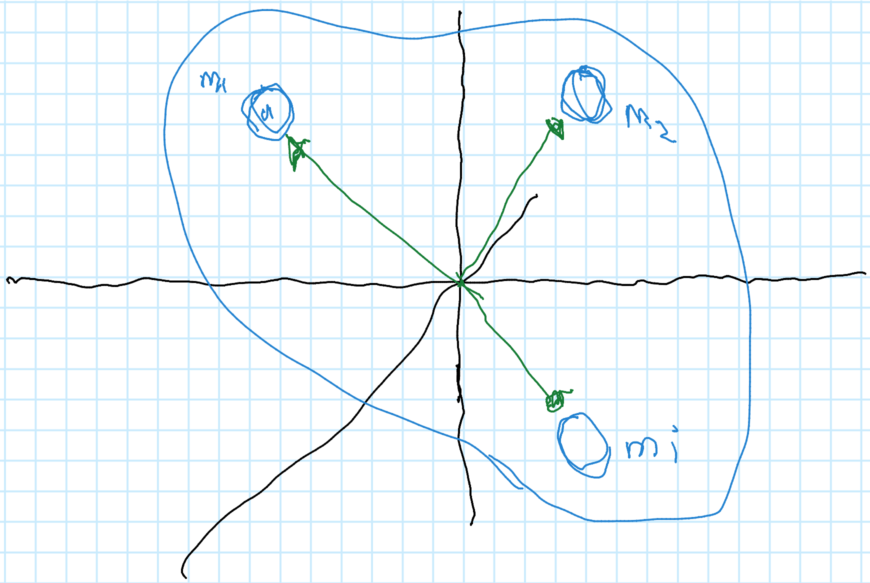


Si $\vec F = \vec{cte}$

$$\vec I = \vec F \Delta t \qquad \vec I_{NETA} = \int_0^t \sum \vec F_{NETO} \, dt$$

$$I_{NETO\,x} = \Delta p_x \,, \quad I_{NETO\,y} = \Delta p_y \,, \quad I_{NETO\,z} = \Delta p_z$$

sistema de particulos



$p_1 = m_1 \vec{V_1}$ $\quad p_2 = m_2 \vec{v}^2$ $\quad \vec{p}_3 = m_3 V_3$

$\vec{P}_{s} = P_1 + P_2 + P_3, \ldots = P_4 = \sum_i m_i V_i$

$\vec{I}_{inicio} = \Delta p_{sistema}$.

Principio de conservación de la cant. mov lineal

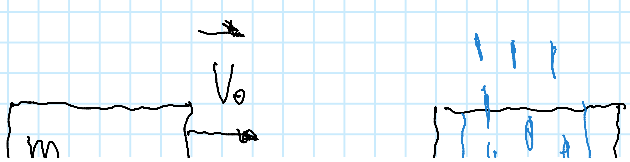
$\Delta \vec{p}$

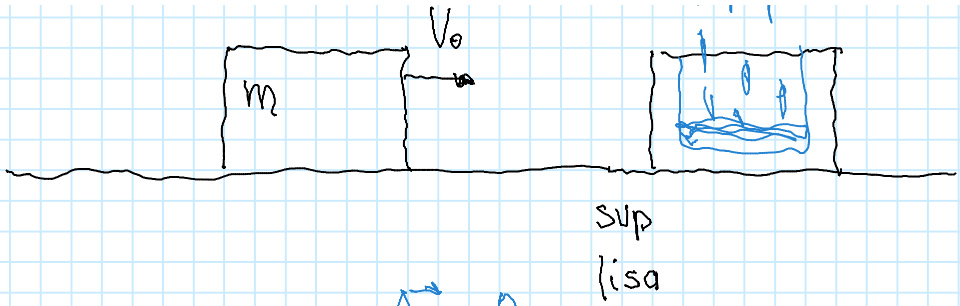
$\vec{p_f} = \vec{p_0}$ sistema aislado $= \vec{cte}$

sistema aislado

↳ colisión

↳ explosión





$$\Delta\vec{p} = 0$$
$$\vec{p}_f = \vec{p}_0$$
$$m_f V_f = m_0 V_0$$
$$m_f > m_0$$
$$V_f < V_0$$

1era ley $\sum \vec{F} = 0$ $\quad \vec{V} = cte \begin{cases} 0 \\ \neq 0 \end{cases}$ $\quad \Delta p = 0$

$\Delta p = cte$

2da ley $\sum \vec{F} = m\vec{a}$, $\sum \vec{F} = \frac{d\vec{p}}{dt}$ $\quad I_{NETO} = \Delta\vec{p}$

3ra ley Ⓐ — Ⓑ $\longrightarrow$ S.A. A y B

$$|I_{A/B}| = |I_{B/A}|$$

$I_{NETO_{A/B}} = \Delta\vec{p}_B$

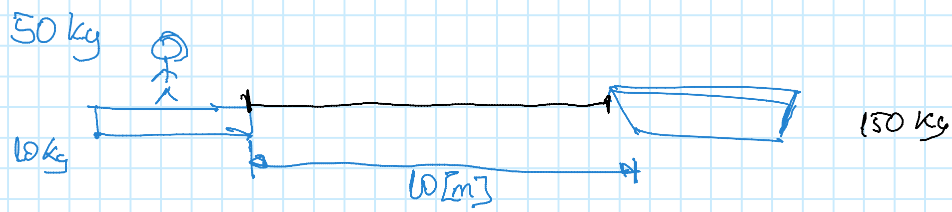
$I_{NETO_{B/A}} = \Delta\vec{p}_A$

$$\mu_{I_{A/B}} = -\mu_{I_{B/A}}$$

$$\Delta\vec{p}_A = -\Delta\vec{p}_B$$
$$\Delta\vec{p}_A + \Delta\vec{p}_B = \vec{0}$$

Δ

Un niño de 50 kg se encuentra sobre una balsa de 10 kg mediante una cuerda le da un jalón a un bote 150 kg mediante una cuerda le da un jalón a un bote 150 kg que se encuentra a 10 [m] si todos los cuerpos se encuentran en reposo. Determine que distancia recorren el niño sobre la balsa y el bote hasta que se encuentran



$$m_A V_{0A} + m_B V_{0B} = m_A V_{fA} + m_B V_{fB}$$

S: N-b y B, S.A.

$$\vec{\Delta p} = 0$$

$$\vec{p}_f = \cancel{\vec{p}_0}\ 0$$

despues antes

$$m_{Nb} V_{Nb} + m_B V_B = 0$$

$$m_{Nb} \cdot \Delta\vec{r}_{Nb} + m_B \Delta\vec{r}_B = 0$$

$$60(\vec{i}) + 150(-\vec{i}) = 0\vec{i}$$

$$60\vec{i} = 150\, d_B \vec{i}$$

$$60\, d_{NB} = d_B\, 150$$

$$\begin{cases} 60\, d_{NB} - 150\, d_B = 0 \\ d_{NB} + d_B = 10 \end{cases}$$

$$d_{NB} = 7.14 [m]$$

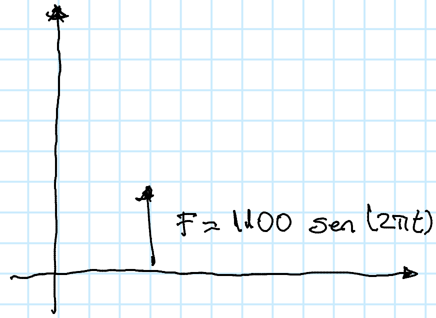
$$d_B = 2.857 [m]$$

$h = 1,9\,m$

$m = 75\,kg$

$t = 0.5\ [s]$

$F = 1100\ sen(2\pi t)$



$I_{neto} = \Delta\vec{p}$

$$\int_{t_0}^{t} F\,dt = P_f - \cancel{P_0}^{\,0}$$

$$\int_0^{0.5} 1100\ sen(2\pi t) = m.V_f$$

$$1100 \int_0^{0.5} sen(2\pi t).\,dt = m.V_f$$

$u = 2\pi.t$

$du = 2\pi\ dt$

$dt = \frac{du}{2\pi}$

$$\frac{1100}{75}\cdot -\frac{\cos(2\pi)}{2\pi}\Big|_0^t$$

$$\frac{1100}{75}\left(1 - \cos(2\pi t)\right)\Big|_0^{0.5}$$

$V = 4.66\ m/s$

$\cancel{V_f}^{\,0} = V_0^2 + 2\Delta x g$

$-\frac{4.66^2}{2g} = \Delta x$

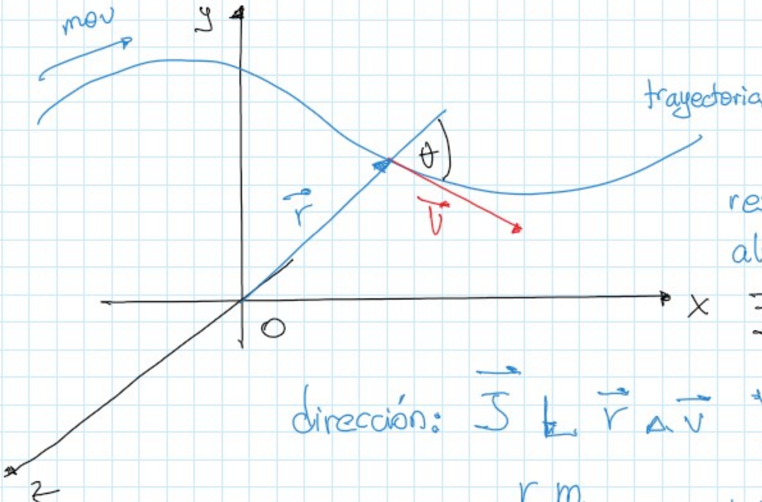
$\Delta x = 1.1079\ [m]$

$\Delta_{max} = 3.00\ [m]$

Momento Angular y Fuerzas Centrales

Monday, November 26, 2018 4:09 PM

Cantidad de movimiento angular asociada al movimiento lineal (traslacional)



respecto de un centro de giro
alrededor de O

$\vec{J} = m[\vec{r} \times \vec{v}]$

dirección: $\vec{J} \perp \vec{r} \wedge \vec{v}$ vector perpendicular al plano formado por $\vec{v}$ y $\vec{r}$

r m

magnitud $|\vec{J}|$ $J = mrv \operatorname{sen}\theta$

$\vec{J} = \vec{r} \times (m\vec{v})$

$\vec{J} = \vec{r} \times \vec{p}$

velocidad y posición
no colineales
paralelas

$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

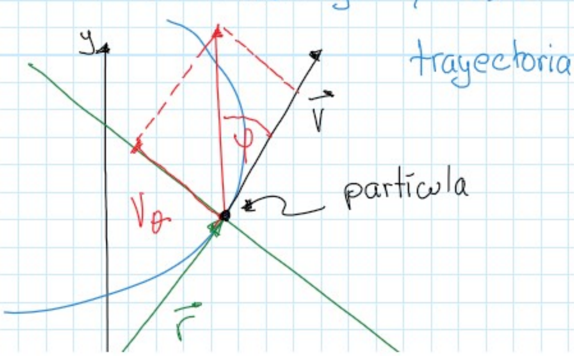
$\vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$

$$\vec{J} = m\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ v_x & v_y & v_z \end{vmatrix} = m\left[(v_z y - v_y z)\vec{i} - (v_z x - v_x z)\vec{j} + (v_y x - v_x y)\vec{k} \right]$$

Plano xy

$\vec{J} = (xv_y - yv_x)\vec{k}$

$\vec{J} = m(xv_y - yv_x)\vec{k}$



$J = mrv \operatorname{sen}\varphi$

$J = mrV_\theta$

$J = mr.r.\dot{\theta}$

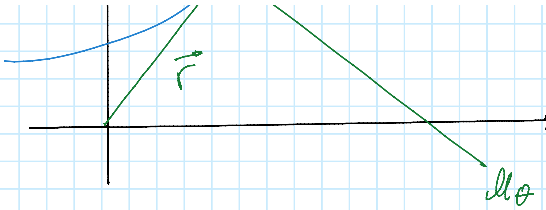
$J = mr^2\dot{\theta}$

alrededor de O

$\vec{J} = m[\vec{r} \times \vec{v}]$

$\dfrac{d\vec{J}}{dt} = m\dfrac{d[\vec{r} \times \vec{v}]}{dt}$

$= m\left[\dfrac{d\vec{r}}{} \times \vec{v} + \dfrac{r \times d\vec{v}}{} \right]$



$J = mr.r.\dot{\theta}$ $\qquad$ dt $\qquad$ dt

$J = mr^2\dot{\theta}$

cantidad de movimiento del radio vector

$$= m\left[\frac{d\vec{r}\times\vec{v}}{dt} + \frac{r\times d\vec{v}}{dt}\right]$$

$$= m\left[\vec{v}\times\vec{v} + r\times\vec{a}\right]$$

$$= m\left[\vec{r}\times\vec{a}\right]$$

$$= m\left[\vec{r}\times\vec{a}\right]$$

$$= \vec{r}\times m\vec{a}$$

$$= \vec{r}\times\sum_i\vec{F}$$

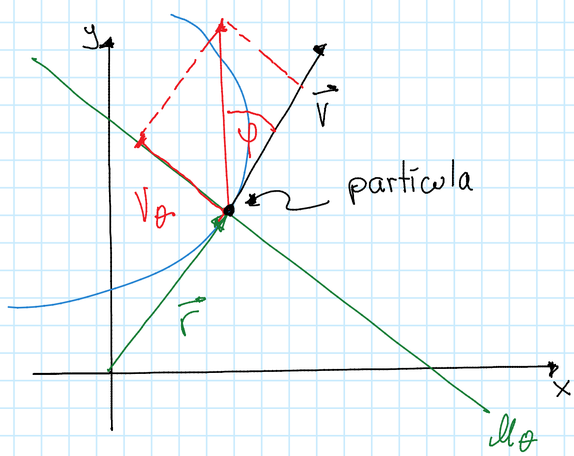
$$\vec{\tau}_1 = \vec{r}\times\vec{F}_1 \qquad \sum_i\vec{\tau} = \vec{r}\times(\vec{F}_1+\vec{F}_2+\ldots)$$

$$\vec{\tau}_2 = \vec{r}\times\vec{F}_2 \qquad \sum_i\vec{\tau} = \vec{r}\times\sum_i\vec{F}$$

torque o momento de una fuerza $\qquad$ ↑ derivada

$$\int_{t_0}^{t}\sum\vec{\tau}\,dt = \int_{J_0}^{J} d\vec{J}$$

$$I_{A_{NETO}} = \Delta\vec{J}$$

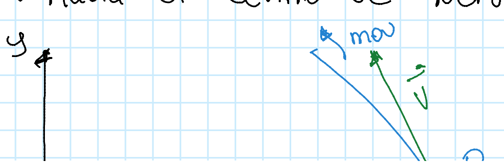


Movimiento bajo la acción de fuerzas centrales.

conservación de la cantidad de movimiento angular.

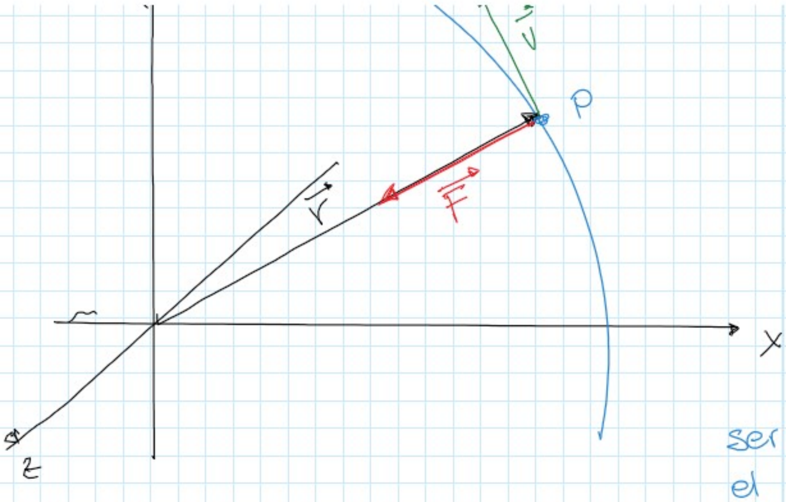
Fuerzas centrales

↳ hacia el centro de fuerza



$$\vec{r}\times\vec{F} = 0$$

$$\Delta\vec{J} = 0$$



$\vec r \times \vec F = 0$

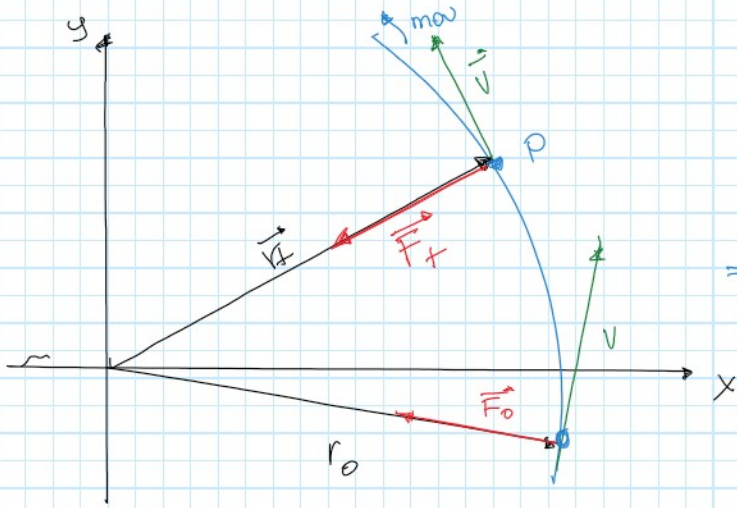
$\Delta \vec J = 0$

$J_f = J_0$

$J_f \vec u_f = J_0 \vec u_0$

$\vec u_f = \vec u_0$

el movimiento debe ser necesariamente en el plano



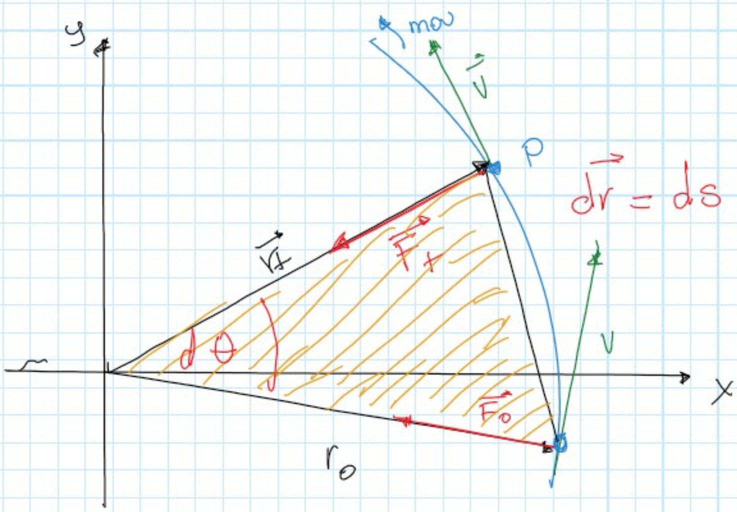
$J_f = J_0$

$m v_f r_f \, sen\, \varphi_1 = m v_0 v_0 \, sen\, \varphi_1$

$= m v_f r_f \, sen\, \varphi_2 = m v_0 r_0 \, sen\, \varphi_1$

$= m r_0^2 \dot\theta_0 = m r_f^2 \dot\theta_f$

$= m r_f^2 \dot\theta_f = m r_0^2 \dot\theta_0$



$d\vec r = ds$

$dA = \frac{1}{2} r \, ds$

$dA = \frac{1}{2} r^2 . d\theta$

$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$

$\frac{dA}{dt} = \frac{1}{2} r^2 . \dot\theta = \frac{1}{2} \frac{J}{m}$

$= \frac{dA}{dt} = \frac{1}{2} . \frac{\vec J}{m} \to cte$

Demostración de la segunda ley de Kepler conservación de la cantidad de movimiento angular

Ejercicios

La posición de un cuerpo $m = 0.5$ kg está dada por $\vec{r} = t^2\vec{i} - t^3\vec{j} + 2t\vec{k}$

determine para $t = 3[s]$

Fuerza neta que actúa sobre el cuerpo

Cantidad de movimiento lineal

Cantidad de movimiento angular

$$\sum \vec{F} = m.\vec{a}$$

$$= 0.5(2\vec{i} - 18\vec{j})$$

$$= \vec{i} - 9\vec{j}$$

$$\vec{P} = m\vec{v}$$

$$\vec{P} = 0.5\,(6\vec{i} - 27\vec{j} + 2\vec{k})$$

$$\vec{P} = 3\vec{i} - 13.5\vec{j} + \vec{k}$$

$$\vec{r}(3) = 9\vec{i} - 81\vec{j} + 6\vec{k}$$

$$\vec{v} = 2t - 3t^2 + 2$$

$$\vec{v}(3) = 6\vec{i} - 27\vec{j} + 2\vec{k} \quad [m/s]$$

$$\vec{a} = 2\vec{i} - 6t\vec{j}\ \Big|_{t=3} = 2\vec{i} - 18\vec{j}$$

$$\vec{J} = m(\vec{r} \times \vec{v})$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 9 & -81 & 6 \\ 2 & -3 & 2 \end{vmatrix} = -1.5\,(48\vec{i} + 2\vec{j} - 45\vec{k})$$

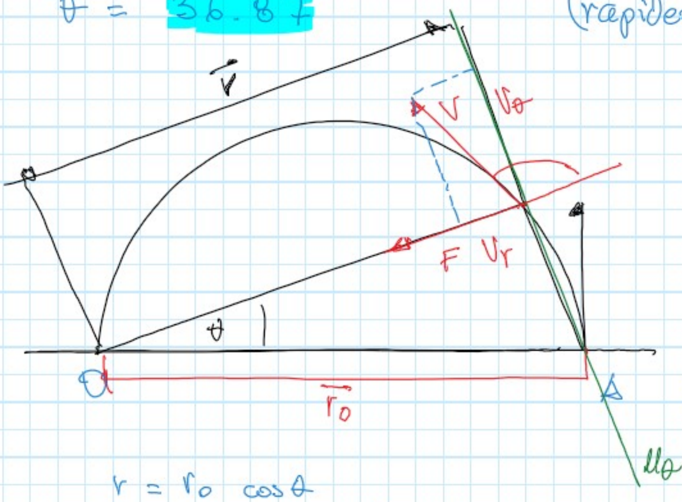
(2)

Una partícula de 2 Kg se lanza desde el punto A, perpendicular

mente a la línea OA con rapidez $V_0 = 3m/s$ y se mueve bajo

Una partícula de 2 Kg se lanza desde el punto A, perpendicularmente a la línea OA con rapidez $V_0 = 3 m/s$ y se mueve bajo la acción de una fuerza central neta $\vec{F}$ dirigida hacia el punto O a lo largo de una trayectoria semicircular donde $r_0 = 4m$.

$\theta = 36.87$ (rapidez y fuerza)



$\Delta \vec{J} = 0$

$\vec{J_0} = \vec{J_f}$

$m r_B V_B sen(\gamma_B) = m r_A V_A \cancel{sen(90°)}$ 1

$2(3.2) V_\theta = 2 \times 4 \times 3$

$= 24 \left[\frac{kg\, m^2}{2}\right]$

$V_\theta = 3.5 = r\dot{\theta}$

$\dot{\theta} = 1.17\ rad/s$

$r = r_0 \cos\theta$

$\dot{r} = r_0 - sen(\theta)\,\dot{\theta}$

$\ddot{r} = r_0 - sen(\theta)\,\ddot{\theta}$

$V_r = r\dot{\theta}$

$V_r = -2.80\ rad/s$

$\vec{V} = V_\theta + V_r$

$\vec{U} = 1.17 + 2.80$

$|\vec{U}| = 3.034\ m/s$

$\sum \vec{F} = m\vec{a}$

$= m(\vec{a_r} + \vec{a_\theta})$

$= m\vec{a_r} + m\vec{a_\theta}$

$= \sum F_r = ma_r + \cancel{\sum F = ma_\theta}$ 0

(9.28) ¿?

$a_r = \ddot{r} - r\dot{\theta}^2$

$a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta}$ → no tiene componente transversal

$0 = 2\dot{r}\dot{\theta} + r\ddot{\theta}$

$-2\dot{r}\dot{\theta} = r\ddot{\theta}$

$-\frac{2(-2.8)(1.17)}{3.2} = \ddot{\theta} = 2.047\ rad/s^2$

$\dot{r} = -V_0\, sen(\theta)\,\dot{\theta}$

$\ddot{r} = -V_0\left(sen(\theta).\ddot{\theta} + \cos(\theta).\dot{\theta}^2\right)$

$\ddot{r} = -V_0\left(sen(\theta)\,\dot{\theta} + \cos(\theta).\ddot{\theta}^2\right)$

$= -(3)($

$a_r = -13.8$

Energia

Monday, December 17, 2018 4:07 PM

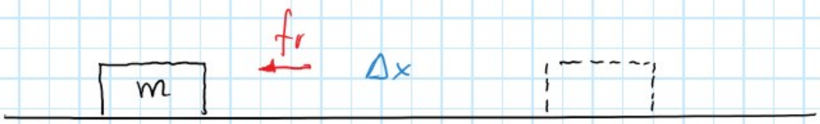
Rotacional $= 0$ Fuerza conservativa

Trabajo realizado por la fuerza de rozamiento T_{Fr}

Tangente a las superficies en contacto

$$T_{fr} = \int_{P_1}^{P_2} \vec{fr} \cdot \vec{dr}$$

$$T_{fr} = -\int_{S_1}^{S_2} fr_T \, dS$$ ← cuando es tangente a la trayectoria } Resistivo



fuerza discipativa

$T_{fr} = -frc \, \Delta x$

Q calor

forma de energia en movimiento

Δ energia interna

$T_{fr} = -Q$

Gráfico (Ep vs x)

sistema masa - resorte

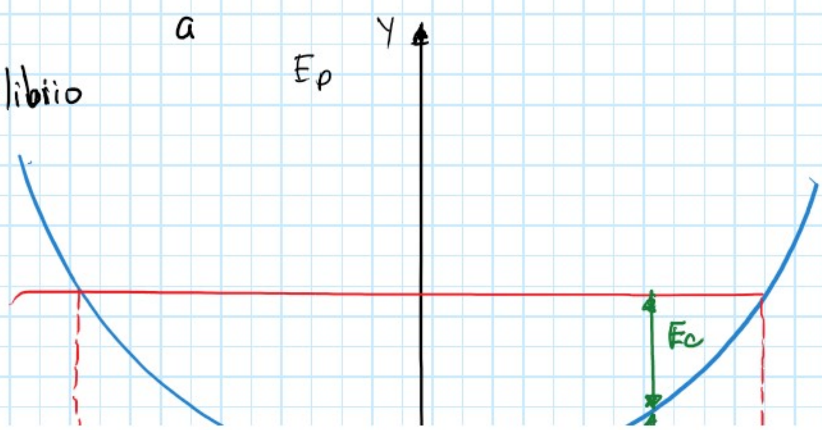
Posición de equilibiio

$\sum F = 0$

$a = 0$

$v = máx$

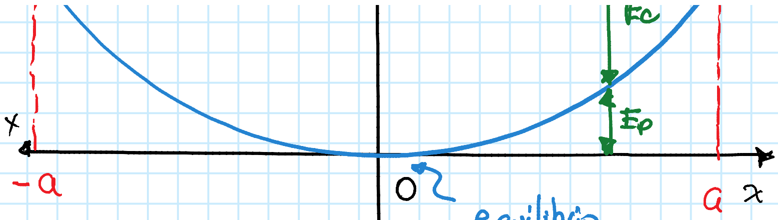
$E_p = \frac{1}{2} k x^2$



$E_p = \frac{1}{2}kx^2$

$P(\theta) = -\frac{1}{2}kx^2$

$F_c = \text{Grad}(\varnothing) = -kx$

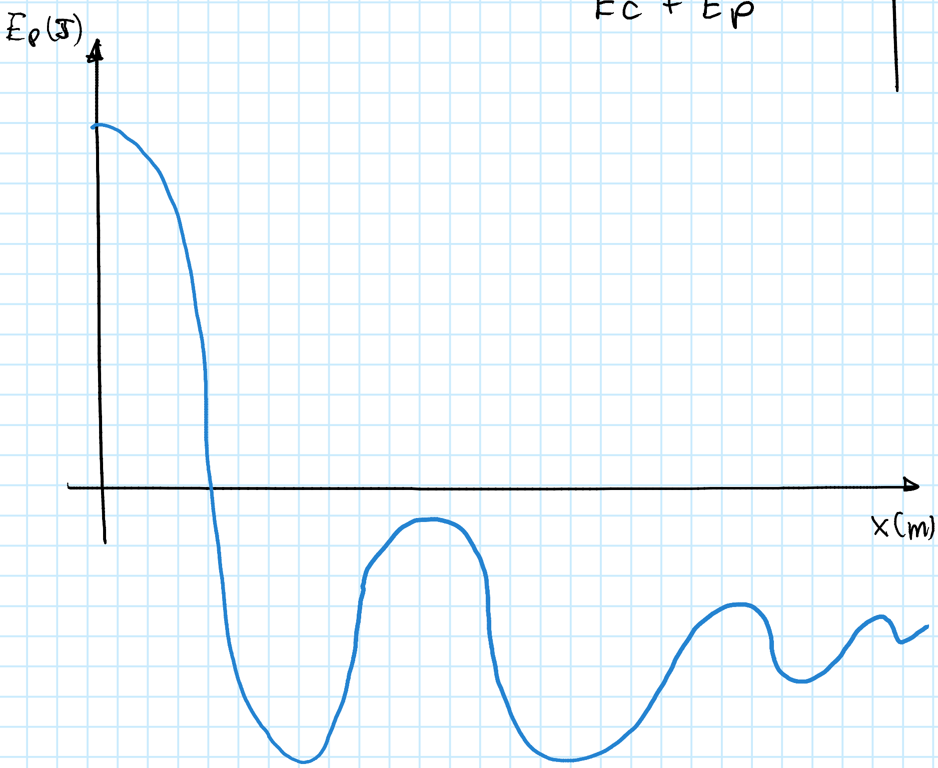


Energía mecánica

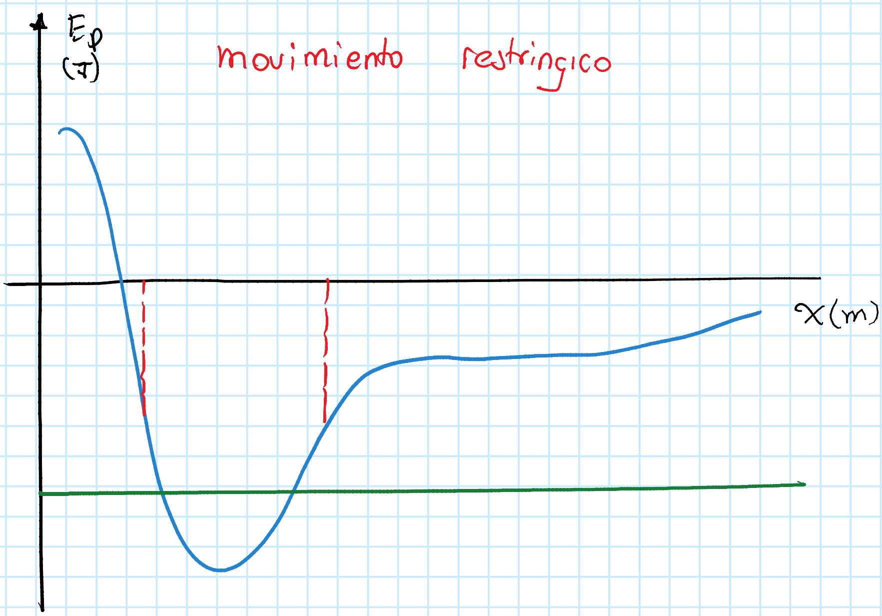
$E_c + E_p$

equilibrio estable

curva alcanza un valor mínimo



movimiento restringico



Equilibrio neutro

$v^2 = \frac{2}{m}\left[E - E_p\right]$ $\quad E \geq E_p$

Equilibrio neutro

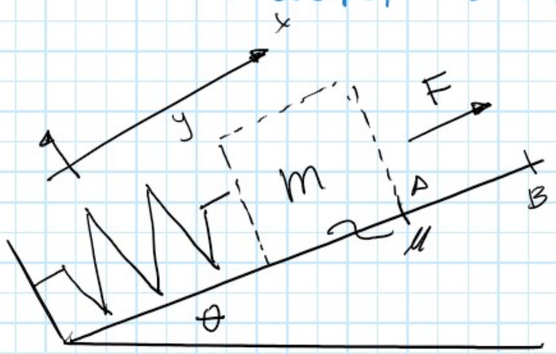
$$E = E_c + E_p$$

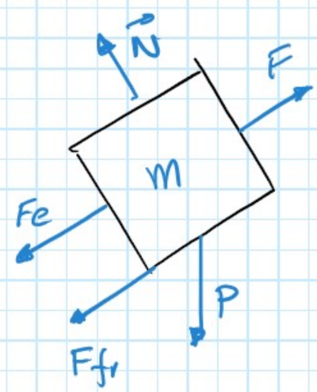
$$E = \frac{1}{2} m v^2 + E_p$$

$$v^2 = \frac{2}{m}\left[E - E_p\right] \quad E \geq E_p$$

Relación general Trabajo y Energía

S: masa cuerpo, resorte, tierra, plano inclinado rugoso





$$\sum_i T_{Fc} + \sum_i F_{NC} = T_N$$

$$(T_p + T_{Fe}) + (T_F + T_{fr}) = T_N$$

$$T_N = \Delta E_c$$

$$T_p = -\Delta E_{pg}$$

$$T_{Fe} = -\Delta E_{pe}$$

$$T_{Ffr} = -Q$$

$$-\Delta E_{pg} - \Delta E_{pe} + T_F - Q = \Delta E_c$$

$$T_F - Q = \Delta E_c + \Delta E_{pg} + \Delta E_{pe}$$

$$T_F = \Delta E_c + \Delta E_{pg} + \Delta E_{pe} + Q$$

$$T_F = \Delta E_c + \Delta E_p + Q$$

$$T_F = \Delta E_M + Q$$

$$T_F + T_{Ffr} = \Delta E$$

$$\sum_i T_{FNC} = \Delta E_M$$

Principio de conservación de la energía mecánica

Si $\sum F_{NC} = 0$

$$E_{M0} + E_{Mf} = E_M = cte$$

Ent...

Si $\sum F_{NC} = 0$

Entonces

$\Delta E_M = 0$

- Solo actuan fuerzas conservativas

$\vec{F}_{NC} = \overset{(+)}{\vec{T}_{activo}} \quad \overset{(-)}{T_{resistivos}} =$

$$\sum_i F = m \cdot a_x$$

$$F = \frac{W}{g} \cdot \frac{dV_x}{dt} \cdot \frac{dx}{dx}$$

$$P = F V = cte.$$

$$F = \frac{P}{V}$$

$$F = \frac{m \, dV_x . V}{dx}$$

$$F = \frac{W}{g} \cdot \frac{dV_x . V}{dx}$$

$$\frac{P}{V_x} = \frac{W}{g} \frac{dV_x V_x}{dx}$$

$$dx = \frac{W \, dV_x \, V_x^2}{Pg}$$

$$dx = \frac{W \, V_x^2 \, dV_x}{Pg}$$

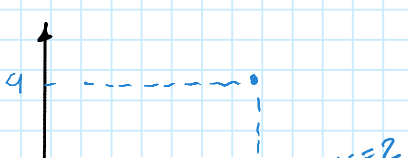
$$\int_0^S dx = \frac{W}{Pg} \cdot \int_{V_1}^{V_2} V_x^2 \, dV_x$$

$$S = \frac{W}{Pg} \left[(V_2)^3/3 - (V_1)^3/3 \right]$$

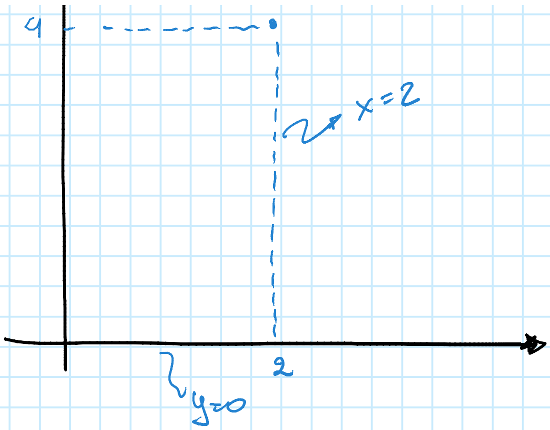
$$S = \frac{W}{3Pg} \left[V_2^3 - V_1^3 \right]$$

$$\vec{F} = (y^2 - x^2)\vec{i} + (3xy)\vec{j}$$

$P_1(0,0) \rightarrow P_2(2,4)$



$$T_F = \int_{P_1}^{P_2} F_1 \, d\vec{r}$$



$$TF = \int_{P_1} F \, dr$$

$$TF = \int_{P_1}^{P_2} F_x \, dx + F_y \, dy$$

$$TF = \int_{P_1}^{P_2} (y^2 - x^2) \, dx + \int_{P_1}^{P_2} 3xy$$

$$TF = \int_0^2 (y^2 - x^2) \, dx + \int_0^4 3xy$$

$$TF = \int_0^2 (0 - x^2) \, dx + \int_0^4 3(2) y$$

$$\int_0^2 x^2 \, dx + \int_0^4 6y$$

$$\frac{8}{3}$$

50.666

Colisiones

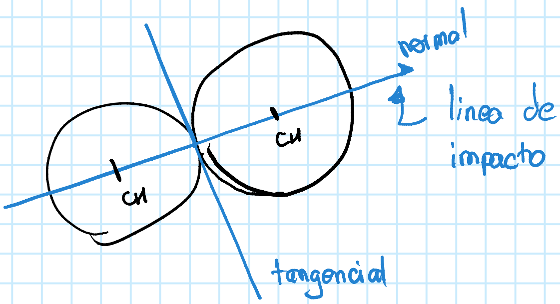
Thursday, January 3, 2019 4:07 PM

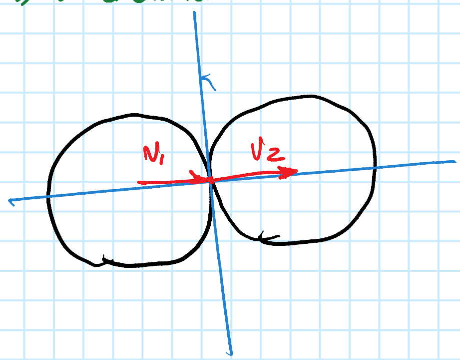
Colisión es

choque - impacto

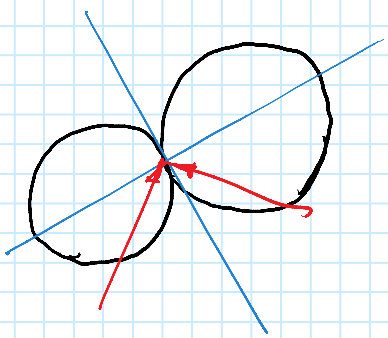
fuerzas impulsivas
intervalo de tiempo pequeño
choque central
centro de masa

línea coincide con la normal
↳ une los centros de masa
↳ exentrico

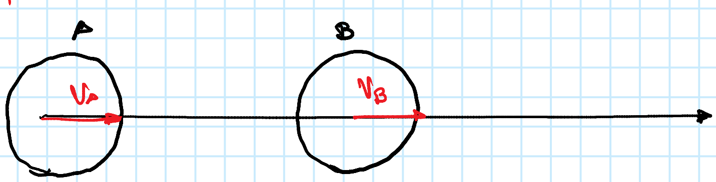




choque central
- directo
- oblicuo



choque central directo



Antes

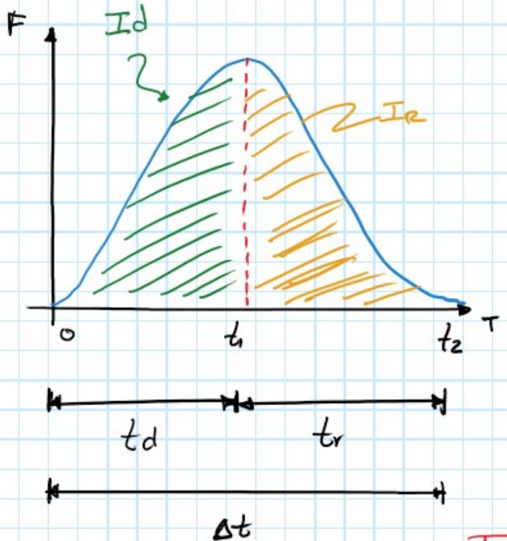
$(V_A) > (V_B)$

Se considera un sistema aislado

$\overrightarrow{\Delta P} = 0$

$\vec{P_f} = \vec{P_0}$

$m_A V_{0A} + m_B V_{0B} = m_A V_{fA} + m_B V_{fB}$



$$e = \frac{\cancel{u} - (V_A)_2 + (V_B)_2 - \cancel{u}}{V_A - \cancel{u} + \cancel{u} - V_B}$$

$$e = \frac{(V_B)_2 - (V_A)_2}{V_A - V_B}$$

caso particular

si $e = 0$

$$(V_B)_2 = (V_A)_2 = V$$

$$\boxed{V' = \frac{m_A V_A + m_B V_B}{m_A + m_B}}$$

si $e = 1$ la colisión es elástica

$$V_A - V_B = (V_B)_2 - (V_A)_2$$

$$m_A (V_A)_2 + m_B (V_B)_2 = m_A V_A + m_B V_B$$

$$m_A \left((V_A)_2 - V_A \right) = m_B \left((V_B)_2 - V_B \right)$$

$$\left((V_A)_2 + V_A \right) = \left((V_B)_2 + V_B \right)$$

$$\frac{1}{2} m_A \left((V_A)_2^2 - V_A^2 \right) = \frac{1}{2} m_B \left((V_B)_2^2 - V_B^2 \right)$$

$$V_A - V_B = (V_B)_2 - (V_A)_2$$

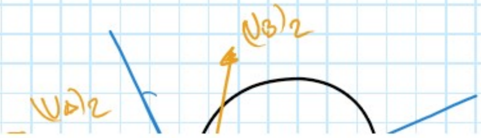
$$V_A + (V_A)_2 = (V_B)_2 + V_B$$

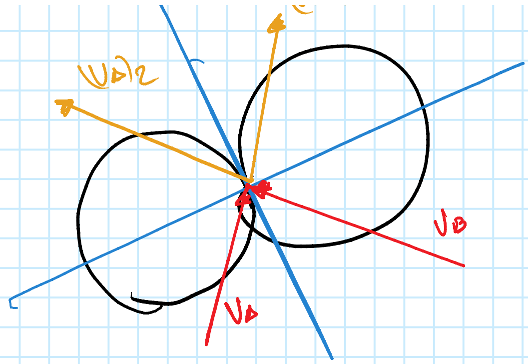
$$Ec_A - (Ec_A)_2 = (Ec_B)_2 - Ec_B$$

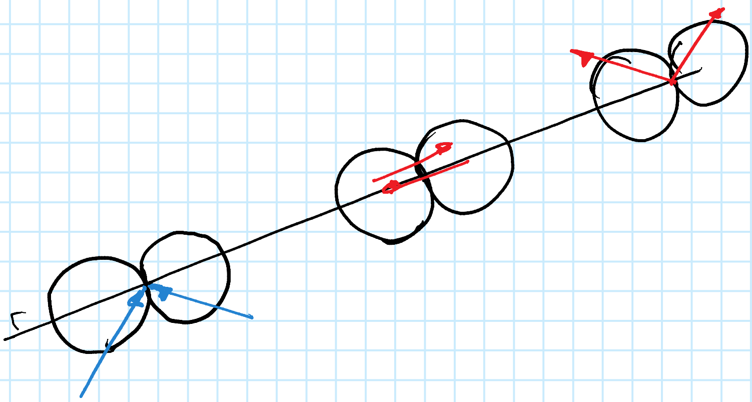
$$Ec_A + Ec_B = (Ec_B)_2 + (Ec_A)_2$$

$$Ec = (Ec)_2 = cte$$

coeficiente de restitución único para cada par de cuerpos







se considera un sistema aislado

$$\Delta\vec{p}=0$$

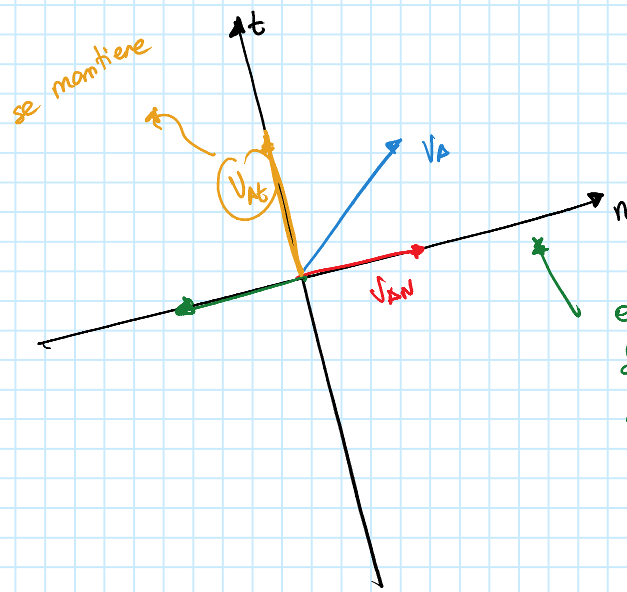
$$\Delta\vec{p_t}+\Delta\vec{p_N}=\vec{0}$$

$$\Delta\vec{p_t}=0 \wedge \Delta\vec{p_N}=0$$

$$\Delta\vec{P_{At}}+\Delta\vec{P_{Bt}}=0$$

deben ser 0

ó iguales en direcciones contrarias



el impulso solo se da en la linea de contacto

$$\Delta\vec{P_{At}}=0$$

$$\Delta\vec{P_{Bt}}=0$$

$$m_A(V_{At})_2 - m_A V_{At}$$

$$(V_{At})_2=(V_{At})$$

↑ en el eje tangencial

$\Delta\vec{P}\ \ = 0$

1

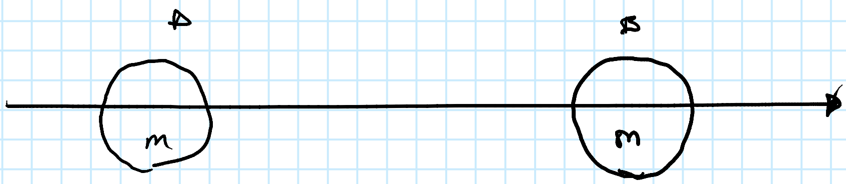
$\Delta \vec{P_N} = 0$

$P_{AN} = (P_{AN})_2$

$m_A (V_A)_2 + m_B (V_B)_2 = m_A V_A + m_B V_B$

$$e = \frac{(V_{BN})^2 - (V_{AN})^2}{V_{AN} - V_{BN}}$$

2 partículas de igual masa se mueven por el eje x una con una velocidad de 0.5 i m/s y otra con una velocidad de -0.2 i m/s, las dos partículas experimentan un choque completamente inelástico, determine el porcentaje de la energía cinética perdida durante la colisión



$$V' = \frac{\cancel{m}(0,5 - 0,2)}{2\cancel{m}}$$

$V' = 0,15 \ m/s$

$\frac{1}{2} \cancel{2}m \ (0,15)$

0.0225

$E_c = \frac{1}{2} m (v)^2 + \frac{1}{2} m (v^2$

$E_c = \frac{1}{2} (0,5) + \frac{1}{2} (0,3)^2$

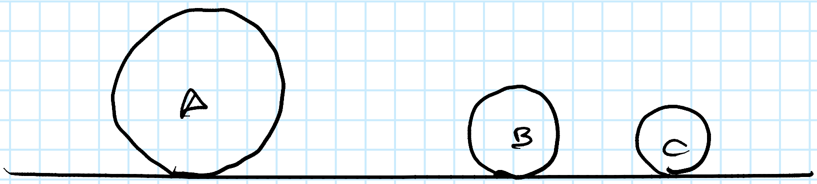
$E_c = m \ (0.145$

$\Delta E_c = -0.1225 \ m$

$$\% \Delta E_c = \frac{-0.1255}{0.145} \cdot 100\%$$

$\% \Delta E_c = 84.48\%$

3 partículas iguales se encuentran en una superficie lisa A = (b) B y C en reposo, si el coeficiente de restitución es e determine la velocidad de c inmediatamente después de ser impactado por B



$$e = \frac{(v_B)_2 - (v_A)_2}{v_A - v_B}$$

$$e\,(v_A - v_B) = (v_B)_2 - (v_A)_2$$

$$(v_B)_2 = e\,(v_A - v_B) + v_A$$

$$(v_B)_2 = e\,(v_A - \cancel{v_B}^{\,0}) + (v_A)_2$$

$$(v_B)_2 = e v_A + (v_A)_2$$

$$e = \frac{(v_C)_2 - (v_B)_2}{v_B - v_C}$$

$$(v_B)_2 + e\,(v_B - v_C) = (v_C)_2$$

$$(v_C)_2 = e\,(v_B - v_C) + (v_B)_2$$

$$(v_C)_2 = e\,\big((e(v_A - v_B) + (v_A)_2) - v_C\big) + (v_B)_2$$

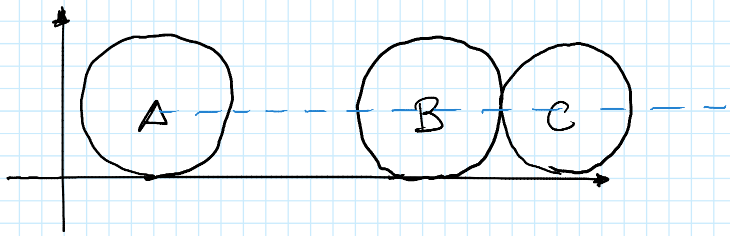
$$(v_C)_2 = e\,\big(e(v_A - \cancel{v_B}) + (v_A)_2 - \cancel{v_C}\big) + (v_B)_2$$

$$(v_C)_2 = e\,(e v_A + (v_A)_2) + (v_B)_2$$

$$(v_C)_2 = e^2 v_A + e(v_A)_2 + (v_B)_2$$

Sistemas de Partículas

Monday, January 7, 2019 4:05 PM



S: A y B

$$m_A(V_A)_2 + m_B(V_B)_2 = m_A V_A + \cancel{m_B V_B}^{0}$$

$$(V_A)_2 + (V_B)_2 = V_A$$

$$e = \frac{(V_B)_2 - (V_A)_2}{V_A - \cancel{V_B}^{0}}$$

$$V_A e = (V_B)_2 - (V_A)_2$$

$$(V_A)_2 + (V_B)_2 = \frac{(V_B)_2 - (V_A)_2}{e}$$

$$2(V_B)_2 = V + Ve$$

$$V_{B_2} = \frac{1}{2} V(1+e)$$

S: B y C

$$\cancel{m_B}(V_B)_2 + \cancel{m_C}(V_C)_2 = \cancel{m_B} V_B + \cancel{m_C V_C}^{0}$$

$$(V_B)_2 + (V_C)_2 = \frac{1}{2} V(1+e) \quad (1)$$

$$e = \frac{(V_C)_2 - (V_B)_2}{V_B - \cancel{V_C}^{0}}$$

$$V_B e = (V_C)_2 - (V_B)_2$$

$$V_C' - V_B' = e \frac{1}{2} V (1+e) \quad (2)$$

$$2V_C' = \frac{1}{2} V (1+e) + \frac{1}{2} Ve(1+e)$$

$$= \frac{1}{2^2} V (1+e)(1+e)$$

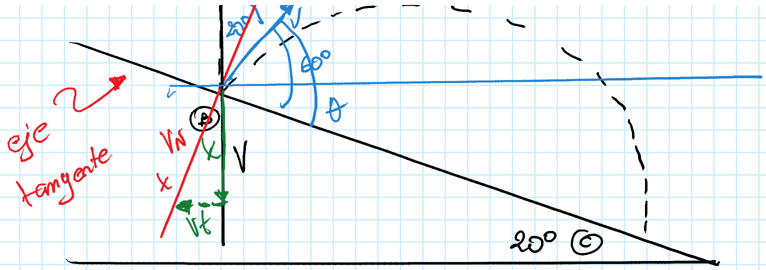
Una pequeña bolita de acero se suelta desde el punto A como se indica en a figura, e impacta en el punto B del plano inclinado liso de la figura. Si $e = 0.85$ determine la distancia BC del rebote de la bolita

$$V_t = 3.84 \, \text{sen}(20°)$$

$$V_t = 1,31 \text{ m/s}$$

$$V_N = 3.84 \cos(20°)$$

$$V_N = 3.61 \text{ m/s}$$



$V_N = 3.84 \cos(20^\circ)$

$V_N = 3.61 \ m/s$

$e = \frac{\cancel{V_P'} - V_b'}{V_b - \cancel{V_P}}$ $\quad e = -\frac{V_b'}{V_b}$

$-V_b' = e V_b$

$-V_b' = 3.067 \ m/s$

$V_f^2 = \cancel{V_0^2}^{0} + 2\Delta x g$

$V_f^2 = 2\Delta x g$

$V_f = \sqrt{2\Delta x g}$

$V_f = \sqrt{2(9.81)(0.75)}$

$V_f = 3.836 \ m/s^2$

$V_{fB} = (0.85)(3.836)$

$V_{fB} = 3.2606$

$V_y = V_0 \ sen(\theta)$

$x_{máx} \ \frac{V_0 \ sen(2\theta)}{g}$

$V = \sqrt{3.067^2 + 1{,}31^2}$

$V = 3.32 \ m/s$

$\tan(\theta) = \frac{3.06}{1{,}31}$

$\theta = 66.82^\circ$

$V_0 = 3{,}33 \ m/s$

$\theta_0 = 46.82^\circ$

$\Delta y = x \tan(\theta) + \frac{g}{2V_0^2 \cos^2(\theta)} x^2$

$= \tan(46.82)x - \frac{9.8 \quad x^2}{2(3.33)^2 + (\cos(46.82))^2}$

$y = -\tan(20^\circ) x$

$0 = \tan(46.82)x + \tan(20^\circ)x - \frac{9.8}{2(3{,}33)^2 (\cos 41{,}82)} x^2$

$0 = x(\tan(20^\circ) + \tan(46.82)) - \frac{9.8}{2(3{,}33)^2(\cos(41{,}82))} x^2$

$x = 1{,}61$

Trabajo y Energía

fuerzas centrales

cantidad de movimiento angular

conservación de la cantidad de movimiento angular

DINÁMICA DE UN SISTEMA DE PARTÍCULAS

Objetivo: Analizar el movimiento de un sistema de partículas

Introducción

sistema de partículas, $\Rightarrow$ conjunto de cuerpos
pueden interactuar entre si y con otros cuerpos
masas definidas (sistema discreto)
n partículas

Cuerpo extenso

posee dimensiones
↳ puede ser considerado un cuerpo extenso
(distribución continua de partículas)
n es infinito

Centro de masa

↳ punto ideal
se encuentra concentrado toda la masa del cuerpo
centro de masa describe el movimiento de todo el cuerpo

Centro geométrico (centroide)

determina la simetría del cuerpo
cuerpo es homogeneo

$$\delta = \text{densidad} = \frac{dm}{dV} = cte$$

coinciden si la distribución es simétrica

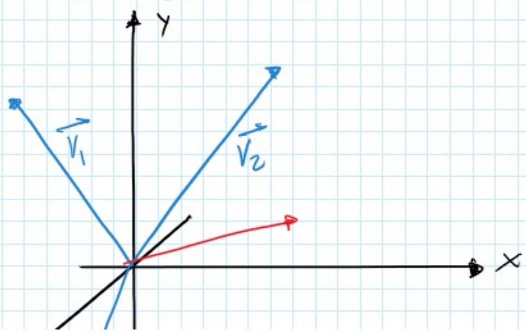
$$CM = CG$$

Centro de gravedad

coinciden
cuando la gravedad es constante

Relaciones fundamentales

Posición del CM $\vec{r}_{CM}$

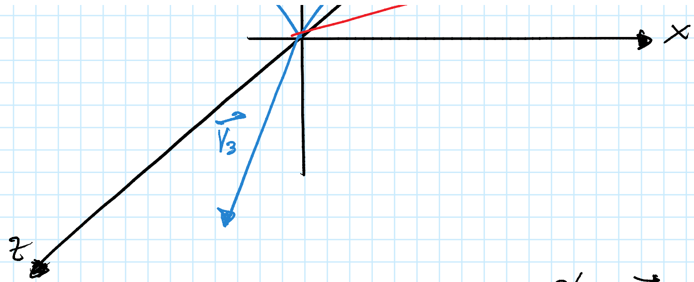


$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \dots + m_n\vec{r}_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$m_1 + m_2 + m_3 + \dots + m_n$

$$\sum_{i=1}^{n} m_i = M$$

n $\rightarrow$



$$\vec{r}_{CM} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{M}$$

$$x_{CM}\vec{i} + y_{CM}\vec{j} + z_{CM}\vec{k}$$

$$= \frac{1}{M} \sum_{i=1}^{n} m_i (x_i\vec{i} + y_i\vec{j} + z\vec{k})$$

① $$x_{CM} = \frac{\sum_{i=1}^{n} m_i x_i}{M}$$

② $$y_{CM} = \frac{\sum_{i=1}^{n} m_i y_i}{M}$$

③ $$z_{CM} = \frac{\sum_{i=1}^{n} m_i z_i}{M}$$

ecuaciones escalares

$$\vec{r}_{CM} = \frac{\int \vec{r}\, dm}{\int dm}$$

velocidad del CM : $\vec{V}_{CM}$

$m_1 V_1 , m_2 V_2 , \ldots , m_n V_n$

$$\vec{r}_{CM} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{n} \qquad \frac{d[\vec{r}_{CM}]}{dt} = \frac{1}{M} \sum_{i=1}^{n} m_i \frac{d[\vec{r}_i]}{dt}$$

$$\vec{V}_{CM} = \frac{\sum_{i=1}^{n} m_i \vec{V}_i}{M}$$

$$\vec{V}_{CM} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2 + \ldots + m_n\vec{V}_n}{M}$$

$$\vec{V}_{CM} = \frac{\vec{P}_1 + \vec{P}_2 + \ldots + \vec{P}_n}{M}$$

$$M V_{CM} = \vec{P}_S$$

Centro de masa

Thursday, January 10, 2019 4:09 PM

$$\vec{V}_{cm} = \frac{\sum_{i=1}^{n} m_i \vec{v}_i}{M}$$

$$M\vec{V}_{cm} = m_1\vec{V}_1 + m_2\vec{V}_2 + \dots$$
$$= \vec{p}_1 + \vec{p}_2 + \dots$$
$$= \vec{P}_s$$

sistema aislado

↳ velocidad constante
puede moverse

Es posible asociar un sistema de referencia al CM

ssi es un sr inercial

Aceleración del centro de masa

$$\vec{a}_{cm} = \frac{d(\vec{V}_{cm})}{dt} = \frac{1}{M}\sum_{i=1}^{n} m_i\left(\frac{d\vec{v}}{dt}\right)$$

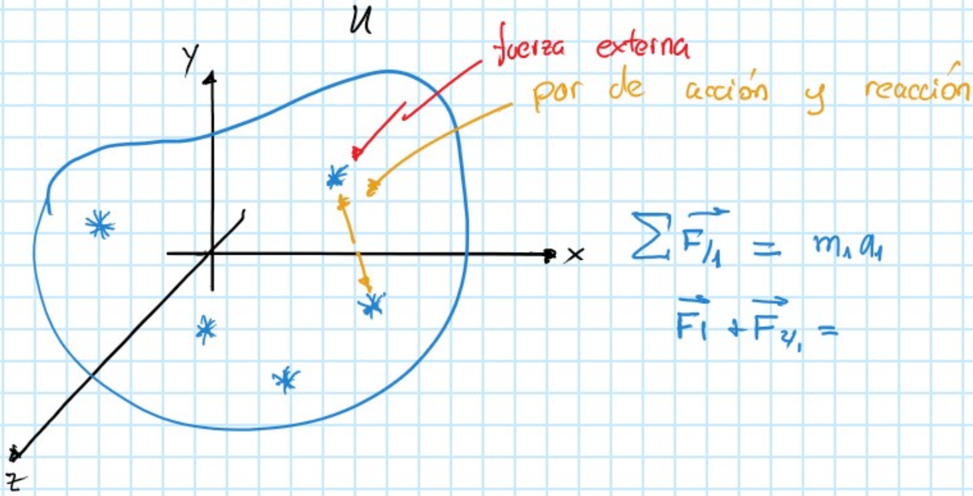
$$\vec{a}_{cm} = \frac{\sum_{i=1}^{n} m_i a_i}{M}$$

$$a_{\vec{cm}} = \sum \vec{F}_{ext/s} = M\,a_{\vec{cm}}$$

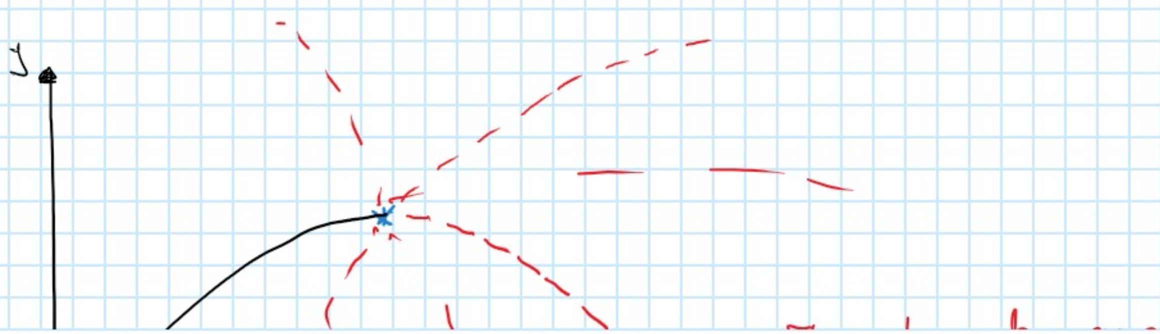
segunda ley de newton para sistemas de partículas

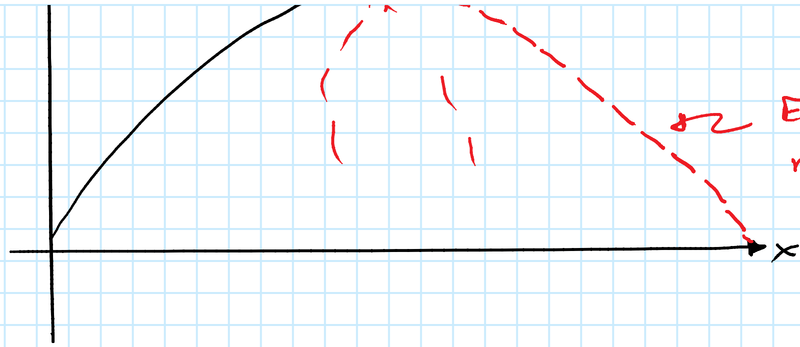
Fuerzas entectivas

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \dots + m_n\vec{a}_n}{M}$$



$$\sum \vec{F}_{/1} = m_1 a_1$$
$$\vec{F}_1 + \vec{F}_{2/1} =$$





El centro de masa mantiene su trayectoria.

Una granada se mueve verticalmente hacia abajo (60 m/s) a 2000 m altura explota en 2 fragmentos iguales. El primero, (80 m/s) hacia abajo, determine después de 10 s de la explosión la posición del centro de masa del sistema

$$V_f = V_0 + at$$
$$= 60 + (9.8)(10)$$
$$= 158$$

$$\vec{\Delta p} = \vec{0}$$
$$p_f = p_0$$
$$\cancel{m}(-80) + \cancel{m}(V_2) = 2\cancel{m}(60)$$
$$V_2 = -40$$

$$V_f^2 = V_0^2 + 2\Delta x g$$

$$\frac{V_f^2 - V_0^2}{2g} = \Delta x$$

$$\Delta x = 1090\,[m]$$

$$h = 2000 - 1090\,[m]$$

$$h_f = 910\,[m]$$ sobre el suelo.

$$y_1 = 2000 - 80(10) + \frac{1}{2}(-9.8)(10^2)$$
$$y_1 = 710\,[m]\ h$$ ✓

$$y_2 = 2000 - 40(10) + \frac{1}{2}(-9.8)(10)^2$$
$$y_2 = 1110\,[m]\ h$$

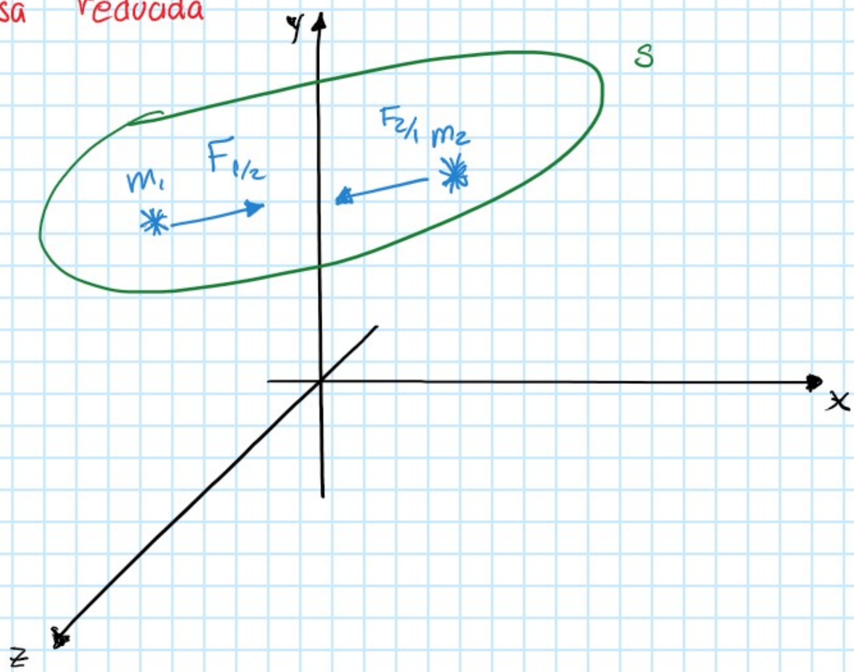
$$y_{cm} = \frac{m(710) + m(1110)}{2}$$
$$y_{cm} = 910$$

$$\sum \vec{F}_{ext/s} = M\,\vec{a}_{cm}$$
$$= M\,\frac{d(v_{cm})}{dt}$$
$$= \frac{d\vec{P_s}}{dt}$$

$$\int_{t_0}^{t_1} \sum F_{ext/s}\,dt = \int_{\vec{P_{s_0}}} \vec{P_s} \Rightarrow I_{NETO/s} = \Delta \vec{P_s}$$

$$\int_{t_0}^{t_1} \sum_i F_{ext/s}\, dt = \int_{\vec{P}_{s_0}}^{\vec{P}_{s_0}} \vec{P}_s \Rightarrow I_{NETO/s} = \Delta \vec{P}_s$$

Masa reducida



$$\sum F = m\vec{a}$$

$$\vec{F}_1 = m_1\vec{a}_1 \qquad \vec{a}_1 = \frac{\vec{F}_1}{m_1}$$

$$-\vec{F}_2 = m_2\vec{a}_2 \qquad \vec{a}_2 = -\frac{\vec{F}_2}{m_2}$$

$$\vec{a}_{1/2} = \vec{a}_1 - \vec{a}_2$$

$$\vec{a}_{1/2} = \vec{F}_1\left(\frac{1}{m_1}\right) - (-\vec{F}_2)\left(\frac{1}{m_2}\right)$$

$$\vec{a}_{1/2} = F\left(\frac{1}{m_1}\right) + F_2\left(\frac{1}{m_2}\right)$$

$$\vec{a}_{1/2} = F\left(\frac{m_2 + m_1}{m_1 m_2}\right)$$

$$\vec{F} = \vec{a}_{1/2}\left(\frac{m_1 \cdot m_2}{m_1 + m_2}\right)$$ ← masa reducida

$$\vec{F} = \vec{a}_{1/2}\,\mu$$

$S: m_1$ y m_2

$\vec{P}_{s/cm}$

$$\vec{V}_{1/cm} = \vec{V}_1 - \vec{V}_{cm}$$

$$= \vec{V}_1 - \left[\frac{\vec{V}_1 m_1 + \vec{V}_2 m_2}{m_1 + m_2}\right]$$

$$= \frac{\cancel{m_1 \vec{V}_1} + m_2 \vec{V}_1 - \cancel{m_1 \vec{V}_1} - m_2 \vec{V}_2}{m_1 + m_2}$$

$$= \frac{m_2 V_1 - m_2 V_2}{m_1 + m_2}$$

$$= \frac{m_2 (\vec{V}_1 - \vec{V}_2)}{m_1 + m_2} = \frac{m_2}{m_1 + m_2} \left(\vec{V}_{1/2}\right)$$

$$\vec{P}_{1/cm} = m_1 \vec{V}_{1/cm}$$

$$= m_1 \frac{m_2}{m_1 + m_2} \vec{V}_{1/2}$$

$$= \mu \vec{V}_{1/2}$$

$$\vec{V}_{2/cm} = \vec{V}_2 - \vec{V}_{cm}$$

$$= \vec{V}_2 - \left[\frac{V_1 m_1 + V_2 m_2}{m_1 + m_2}\right]$$

$$= \frac{m_1 V_2 + \cancel{m_2 V_2} - V_1 m_1 - \cancel{V_2 m_2}}{m_1 + m_2}$$

$$= \frac{m_1 (V_2 - V_1)}{m_1 + m_2}$$

$$= \frac{m_1}{m_1 + m_2} \vec{V}_{2/1}$$

$$\vec{P}_{2/cm} = -m_2 \left(\frac{m_1}{m_1 + m_2}\right) \vec{V}_{2/1}$$

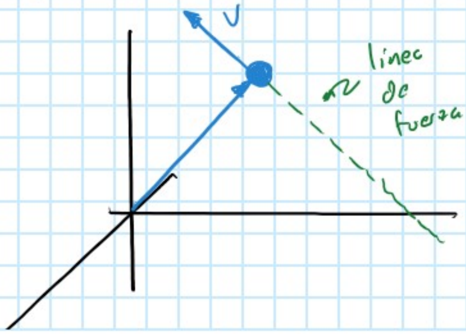
$$\vec{P}_{2/cm} = -\mu \vec{V}_{2/1}$$

sistema de referencia en el centro de masa
sistema de momento 0

la suma es igual a 0

Cantidad de movimiento angular de un sistema de partículas

S: 1 partícula



Alrededor de O

$$\vec{S} = m(\vec{r} \times \vec{v})$$

$$= \vec{r} \times \vec{p}$$

$$\frac{d\vec{S}}{dt} = m\left[\frac{d\vec{r}}{dt} \times \vec{v} + \vec{r} \times \frac{d\vec{r}}{dt}\right]$$

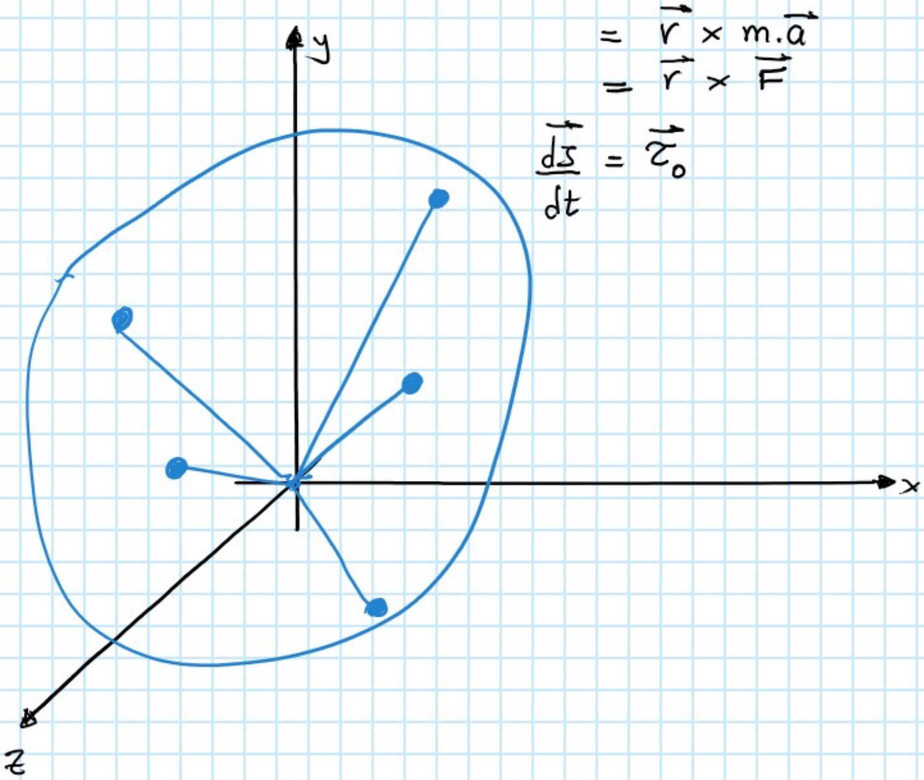
$$\frac{d\vec{J}}{dt} = m\left[\frac{d\vec{r}}{dt}\times\vec{v} + \vec{r}\times\frac{d\vec{r}}{dt}\right]$$

$$= m\left[\vec{r}\times\vec{a}\right]$$

$$= \vec{r}\times m.\vec{a}$$

$$= \vec{r}\times\vec{F}$$

$$\frac{d\vec{J}}{dt} = \vec{\tau}_0$$



Todos los torques interna se anulan

torque cero

cantidad de movimiento angular alrededor del centro de masa

Spin

Alrededor de O

$$\vec{J}_S = \vec{J}_1 + \vec{J}_2 + \vec{J}_3 + \ldots + \vec{J}_n$$

$$= m_1[\vec{r}_1\times\vec{v}_1] + m_2[\vec{r}_2\times v_2]$$

$$\sum_{i=1}^{n} m_1[\vec{r}_1\times\vec{m}_1]$$

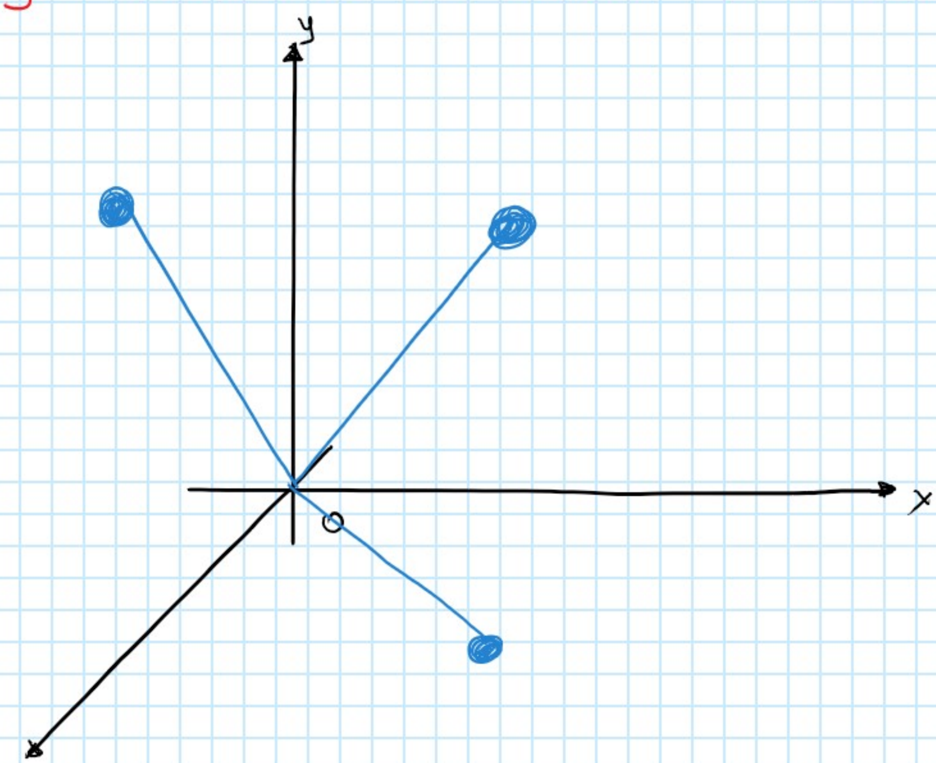
$$d[\vec{J}_S] \quad \sum \vec{\tau}_{ext}$$

$$\frac{d[\vec{J}_s]}{dt} = \sum_{i=1} \vec{\tau}_{ext/s}$$

$$\int_{\vec{J}}^{\vec{J}} d\vec{J}_s = \int_{t_0} \sum_i \tau_{ext/s}\, dt$$

$$\vec{I}_{A\,neto} = J$$

Energia cinética de un sistema de partículas



$$Ec = \frac{1}{2} m_i v_i^2$$

$$Ec_s = \sum_{i=1}^{n} \frac{1}{2} m_i v_i^2$$

Torques internos se anulan

Trabajos internos pueden sumarse

Energía cinética → escalar positivo

$$TN = \Delta Ec$$

$$\sum_1 T_{interno} + \sum_1 {}_{ext} = \Delta Ec$$

$S: m_1 , m_2$

$\vec{V_1} , \vec{V_2}$ ← medidas en un sistema inercial

① $Ec_S = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

② $Ec_S = Ec_1 + Ec_2$

$$Ec_{1/CM} = \frac{1}{2} m (V_{1/CM})^2 , \quad Ec_{2/CM} = \frac{1}{2} (V_2/CM)^2$$

$$V_{CM} = \frac{\sum_{i=1}^{2} m_i V_i}{M}$$ ← velocidad del centro de masa.

$$= \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2}$$

$Ec_1 = \frac{1}{2} m_1 V_1^2$ $\qquad V_{1/CM} = V_1 - V_{CM}$

$\qquad V_1 = V_{1/CM} + V_{CM}$

$= \frac{1}{2} m_1 (V_1 \cdot V_1)^2$

$= \frac{1}{2} m_1 \left[(\vec{V_{1/CM}} + \vec{V_{CM}}) \cdot (\vec{V_{1/CM}} + V_{CM}) \right]$

$Ec_1 = \frac{1}{2} m_1 [(V_{1/CM})^2 + 2 \vec{V_{CM}} \cdot \vec{V_{1/CM}} + V_{CM}^2]$

$$Ec_1 = \frac{1}{2} m_1 \left[(V_{1/CM})^2 + 2 \vec{V}_{CM} \vec{V}_{1/CM} + V_{CM}^2 \right]$$

$$Ec_2 = \frac{1}{2} m_2 \left[(V_{2/CM})^2 + 2 V_{CM} \cdot V_{2/CM} + V_{CM}^2 \right]$$

$$Ec_S = Ec_1 + Ec_2$$

$$Ec = Ec_{S/CM} + \frac{1}{2} V_{CM}^2 (m_1 + m_2) + \left[\vec{V}_{CM} \cdot \vec{V}_{1/CM} + \vec{V}_{CM} \cdot \vec{V}_{2/CM} \right]$$

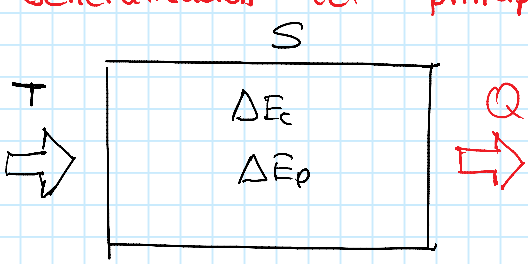
$$Ec = Ec_{S/CM} + \frac{1}{2} V_{CM}^2 (m_1 + m_2) + \left[V_{CM} \cdot (V_1 - V_{CM}) + V_{CM} \cdot (V_2 - V_{CM}) \right]$$

$$Ec_{S/CM} + \frac{1}{2} V_{CM}^2 (m_1 + m_2) + \left[\underbrace{\cancel{m_1 V_{1/CM} + m_2 V_{2/CM}}}_{} \right]^{0} \cdot V_{CM}$$

$$Ec = Ec_{S/CM} + \frac{1}{2} V_{CM}^2 (M)$$

cantidad de movimiento angular
respecto al centro de masa

Generalización del principio de conservación de la energía

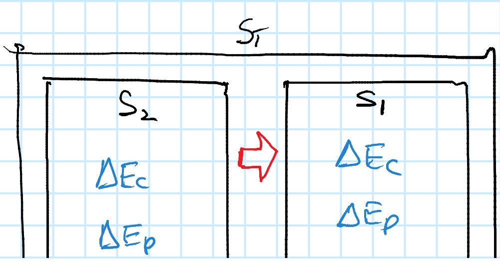


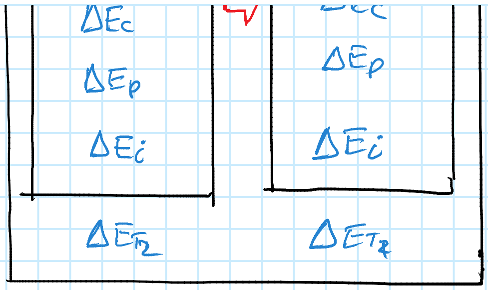
$Q = \Delta E$ interna del sistema

$$T = \Delta EM + Q$$

$$T = \Delta Ec + \Delta Ep + \Delta Ei \; ; \quad E_T = Ec + Ep + Ei$$

$$T = \Delta Ei$$





$$\Delta E_{T_1} = -\Delta E_{T_2}$$

$$\Delta E_{c_1} + \Delta E_{p_1} + \Delta E_{i_1} = -\left(\Delta E_{c_2} + E_{p_2} + E_{i_2}\right)$$

$$\Delta E_{c_1} + \Delta E_{p_1} + \Delta E_{i_1} - \Delta E_{c_2} - E_{p_2} - E_{i_2} = 0$$

$$\left(E_{c_1 f} - E_{c_o}\right) + \left(E_{c_2 f} - E_{c_{1o}}\right) = 0 \quad ¿?$$

$$E_{cf_{ST}} - E_{co_{ST}} = 0$$

$$\Delta E_{c_{ST}} + \Delta E_{p_{ST}} + \ldots + \Delta E_{i_{ST}} = 0$$

$$\Delta E_{T_{ST}} = 0 \quad \checkmark$$

Mecánica del Sólido

Monday, January 14, 2019 4:08 PM

Mecánica del sólido

Objetivo: Conceptos relacionados al sólido

Cuerpo rígido, sólido

Leyes de la mecánica newtoniana

Cuerpo rígido, sólido ⇒

cuerpo extenso

masa distribuido

sistema de partículas

n partículas $n \to \infty$

no se deforma ante la aplicación de una fuerza

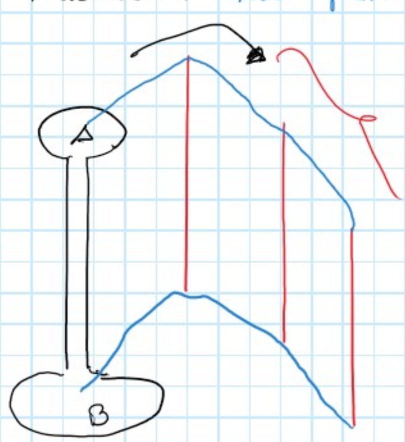
Las posiciones relativas de los partículas no cambian

Sistema deformable, las posiciones cambian

↳ sistema masa - resorte

dos tipos de movimiento

↳ traslación: Las partículas describen trayectorias paralelas.

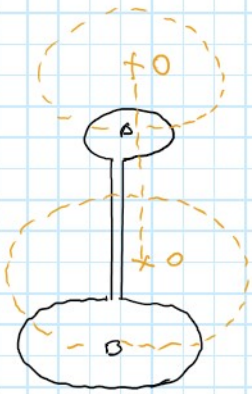


la orientación de la recta no cambia

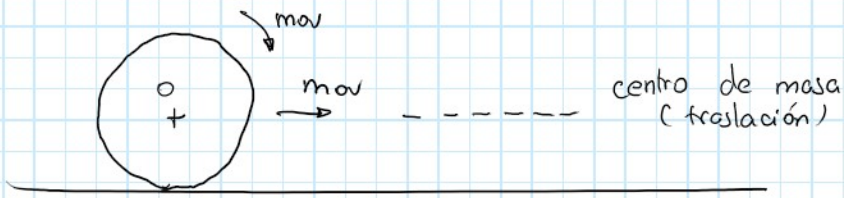
↳ rotación Las trayectorias describen circunferencias

↳ rotación Las trayectorias describen circunferencias

¡rotación pura, el eje no cambia!



Movimiento general (rodadura o rodamiento)



$$\sum \vec{F}_{/S} = M\,\vec{a}_{cm}$$

$$\sum \vec{F}_{/S} = \frac{d\vec{p}}{dt}$$

$$x_{cm} = \frac{\int x\,dm}{\int dm} \qquad \rho = \frac{dm}{dV}$$

CENTRO DE MASA

$$\vec{r}_{cm} = \frac{\sum_{i=1}^{\infty} m_i \vec{r}_i}{\sum_{i=1}^{\infty} m_i}$$

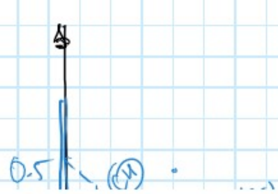
$$y_{cm} = \frac{\int y\,dm}{\int dm}$$

$$z_{cm} = \frac{\int z\,dm}{dm}$$

$$\vec{r}_{cm} = \frac{\int \vec{r}\,dm}{\int dm} \qquad M = \int dM$$

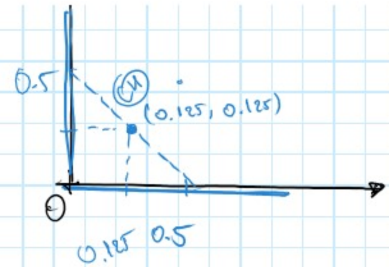
δ constante $\Rightarrow$ centro de masa = centro geométrico (centroide)

Determine $\vec{r}_{cm}$ de una varilla de aluminio que se ha doblado en la mitad formando un ángulo recto



$$X = \frac{0.25\,\frac{m}{2}}{m}$$

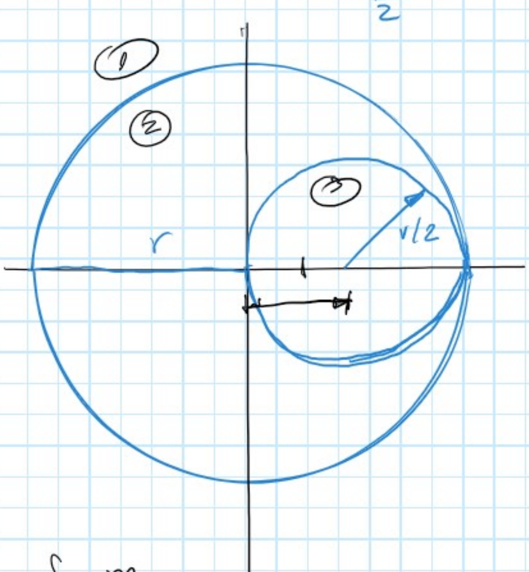
$$y_{cm} = 0.25\,\frac{m}{2}$$



$$y_{cm} = \frac{0.25\,\frac{m}{2}}{m}$$

$$y_{cm} = 0.125$$

Sobre una placa circular homogenea de radio r se realiza un corte circular de radio $\frac{r}{2}$



$$x_{cm1} = x_{cm2} + x_{cm3}$$

① < ②, ③

$$0 = \frac{x_2 m_2 + x_3 m_3}{m_2 + m_3}$$

$$0 = x_2\left(\pi R^2 - \frac{\pi R^2}{4}\right) + \frac{R}{2}\left(\frac{\pi R^2}{4}\right)$$

$$0 = x_2\left(\frac{3\pi R^2}{4}\right) + \frac{R^3\pi}{8}$$

$\delta = \frac{m}{S}$

$m = \delta . S$

① $= \pi R^2$

② $= \pi R^2 - \pi\left(\frac{R}{2}\right)^2$

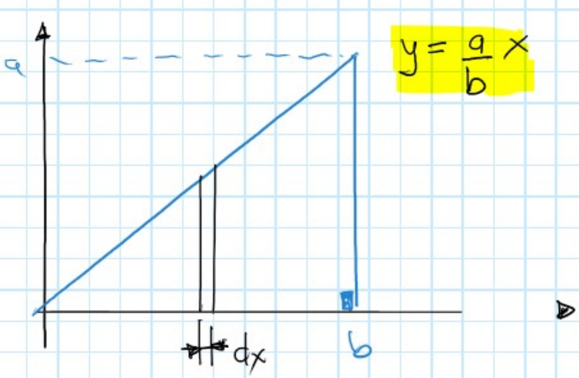
$\pi R^2 - \frac{\pi R^2}{4}$

③ $\frac{\pi R^2}{4}$

$$\frac{-\frac{R^3\pi}{8}}{\frac{3}{4}\pi R^2}$$

$$\boxed{-\frac{R}{6}}$$

Determine el centro de masa



$y = \frac{a}{b}x$

$$\left(\frac{0+b+b}{3}, \frac{0+a}{3}\right)$$

$$\left(\frac{2b}{3}, \frac{a}{3}\right)$$

$$\frac{y}{a} - \frac{x}{b} = 0$$

$dA = y\,dx$

$x_{cm} = \int x\, dm$ $\quad y =$

$dA = y\,dx$

$A = \int_0^b y\,dx$

$A = \int_0^b \frac{a}{b} x\,dx$

$A = \frac{a}{b} \cdot \frac{x^2}{2}\Big]_0^b$

$A = \frac{a}{b}\left(\frac{b^2}{2} - \frac{0}{2}\right)$

$A = \frac{a}{\cancel{b}}\left(\frac{b^{\cancel{2}}}{2}\right)$

$A = \frac{ab}{2}$

$\int x\,dA = \int_0^b x\,y\,dx$

$= \int_0^b x \cdot \frac{a}{b} x\,dx$

$= \int_0^b \frac{a}{b} x^2\,dx$

$= \frac{a}{b}\int_0^b x^2\,dx$

$= \frac{a}{b}\,\frac{x^3}{3}\Big]_0^b$

$= \frac{a}{\cancel{b}}\,\frac{b^{\cancel{3}}}{3}$

$= \frac{ab^2}{3}$

$y_{CM} = \int y\,dA$

$x_{CM} = \frac{\int x\,dm}{\int dm}$ $\quad y =$

$dm = \rho\,dA$

$= \frac{\int x\,dA}{\int dA} = \frac{\int_0^b x\,(y\,dx)}{\frac{1}{2}ab}$

$= x$

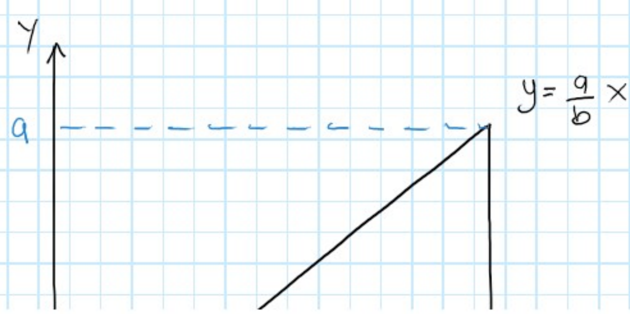
$x_{CM} = \frac{\int x\,dm}{\int dm}$ $\quad \rho = \frac{dm}{dA}$

$dm = \rho\,dA$

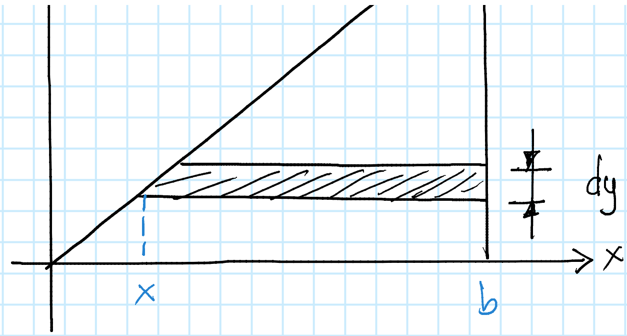
$= \frac{\int x\,dA}{\int dA}$

$\frac{\frac{ab^2}{3}}{\frac{1}{2}ab}$

$x_{CM} = \frac{2b}{3}$



$$y_{CM} = \frac{\int y \, dA}{\int dA}$$



$dA = (b-x)\, dy$

$$\int y(b-x)\, dy$$

$$\int y\left(b - \frac{b}{a}y\right) dy$$

i) $\int_0^a (b-x)\, dy$

$$\int_0^a \left(b - \frac{b}{a}y\right) dy$$

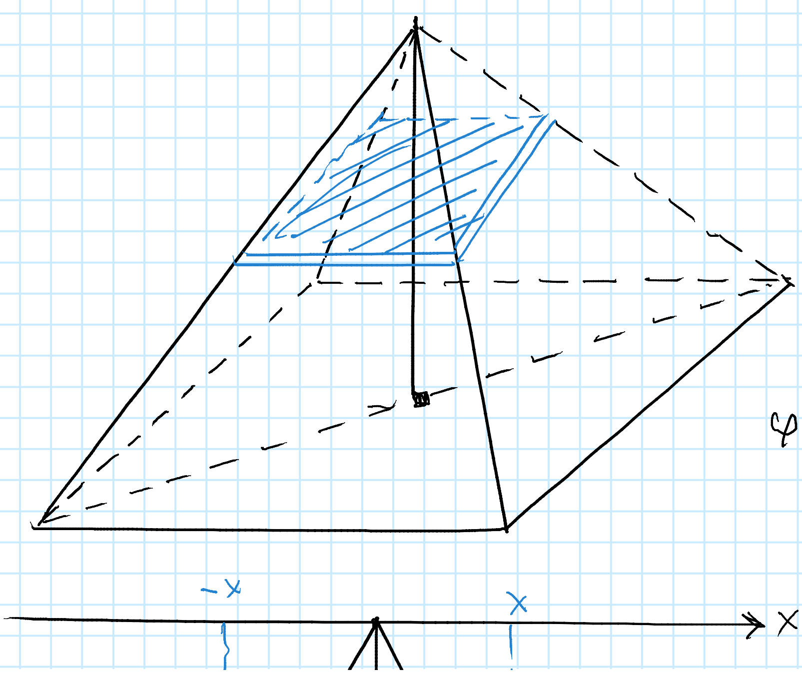
$$\int_0^a b\,dy - \int_0^a \frac{b}{a} y\, dy$$

$$by \Big|_0^a - \frac{by^2}{2a}\Big|_0^a$$

$$ba - \frac{ba^2}{2a}$$

$$ba - \frac{ba}{2} = \frac{ab}{2}$$

A que altura de la base se encuentra el CM de una pirámide cuadrada de altura H

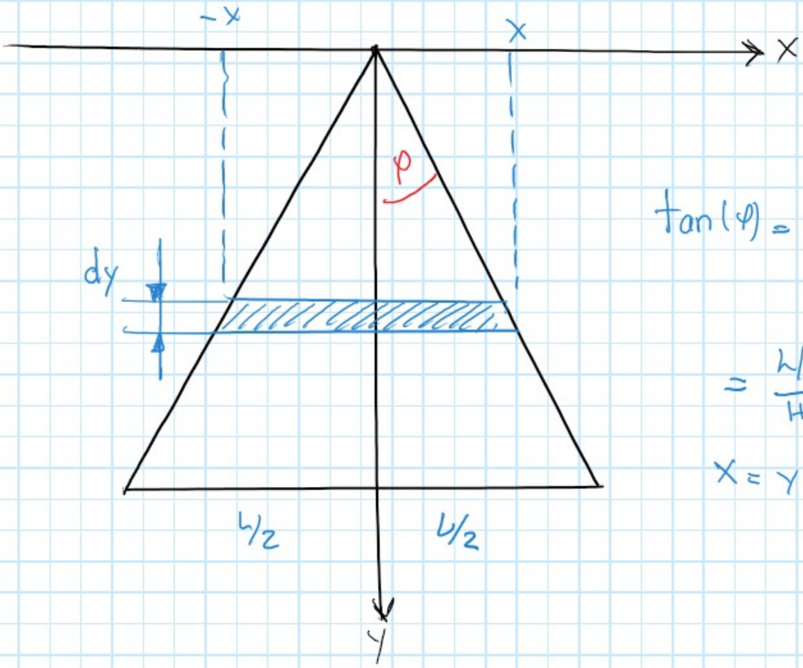


$$y_{CM} = \frac{\rho \int y \, dV}{\rho \int dV}$$

$$\rho = \frac{dm}{dV}$$

$dV = 4x^2 dy$

y_{CM} $\int_0^H y\,(4x^2)\, dy$



$$\tan(\varphi) = \frac{x}{y} = \frac{L/2}{H} = k$$

$$x = yk$$

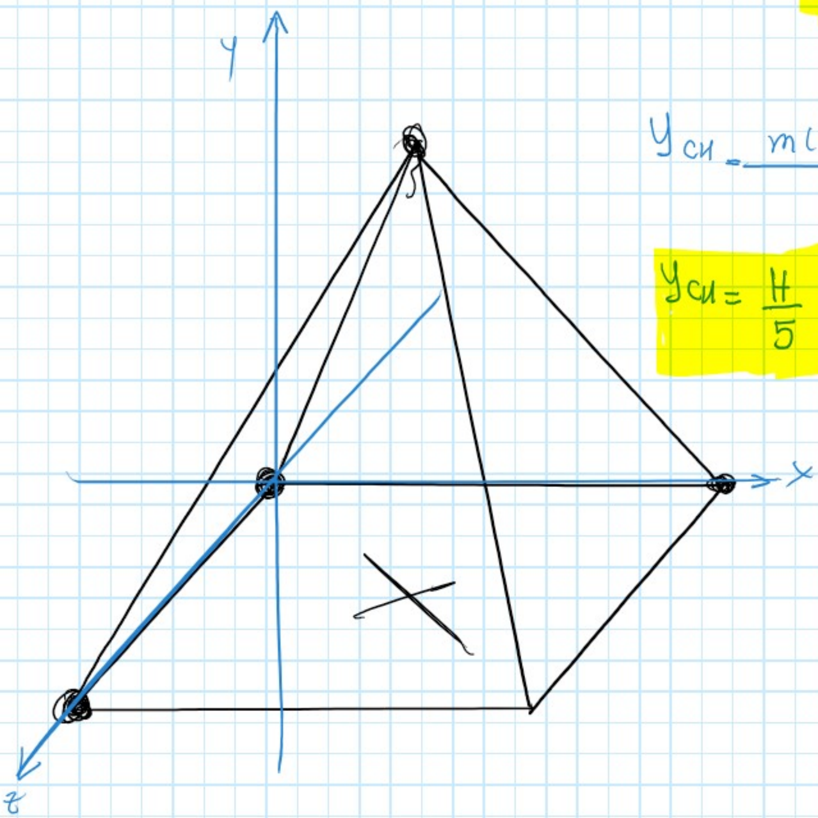
$$Y_{CM} = \frac{\int_0^H Y\,(4x^2)\,dy}{\int_0^H 4x^2\,dy} = \frac{\int_0^H y\,(y\cancel{k})^2\,dy}{\int_0^H (y\cancel{k})^2\,dy} = \frac{\int_0^H y^3\,dy}{\int_0^H y^2\,dy}$$

$$= \frac{\left.\frac{y^4}{4}\right|_0^H}{\left.\frac{y^3}{3}\right|_0^H} = \frac{\frac{H^4}{4}}{\frac{H^3}{3}} = \frac{3}{4}H$$

respecto al vértice

$\Downarrow$

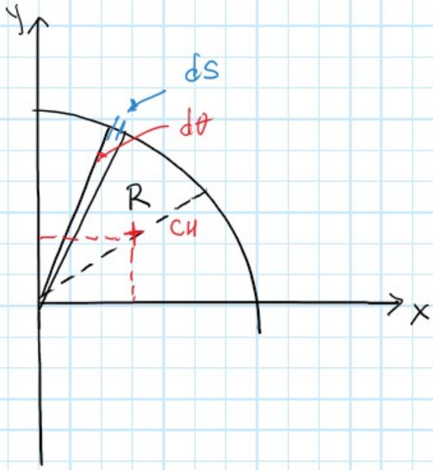
$\frac{H}{4}$ respecto a la base



$$Y_{CM} = \frac{m(0) + m(0) + m(0) + m(0) + mH}{5m}$$

$$Y_{CM} = \frac{H}{5}$$

CM de un alambre uniforme que describe un arco de circunferencia de radio R



$$\rho = \frac{dm}{ds}$$

$$dm = \rho\, ds \qquad ds = R\, d\theta$$

$$y_{CM} = \frac{\int y\, ds}{\int ds} \qquad \int ds = \int_0^{\theta} R\, d\theta$$

$$= R\theta \Big|_0^{\theta = \pi/2}$$

$$\int y\, ds = \int_0^{\theta=\pi/2} R \operatorname{sen}(\theta) R\, d\theta \qquad = \frac{R\pi}{2} \qquad R\theta$$

$$= R^2 \int_0^{\theta=\pi/2} \operatorname{sen}(\theta)$$

$$= R^2 \cos(\theta) \Big|_0^{\pi/2}$$

$$= R^2\,(1 - \cos(\theta))$$

$$\frac{R^2(1-\cos(\theta))}{R\theta}$$

$$\frac{R(1-\cos(\theta))}{\theta}$$

$$= \frac{2R}{\pi}$$

$$x_{CM} = \frac{\int x\, ds}{\int ds}$$

$$\int_0^s x\, ds = \int_0^{\theta} R\cos(\theta) R\, d\theta$$

$$= \int_0^{\theta} R^2 \cos(\theta)\, d\theta$$

$$= R^2 \int_0^{\theta} \cos(\theta)\, d\theta$$

$$= R^2 (\operatorname{sen}(\theta) - \cancel{\operatorname{sen}(0)}^{0})$$

$$= R^2 \operatorname{sen}(\theta)$$

$$x_{CM} = \frac{R^{\cancel{2}} \operatorname{sen}(\theta)}{\cancel{R}\,\theta}$$

$$= \frac{R \operatorname{sen}(\theta)}{\theta}$$

$$= \frac{2R}{\pi}$$

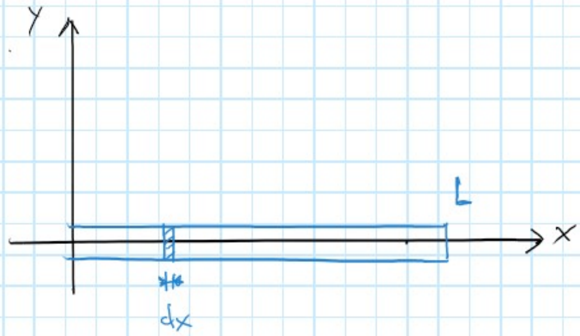
Varilla rígida de longitud L y densidad variable

$$\rho = a\left(1 + \frac{x^2}{L^2}\right)$$

$$x_{CM} = \frac{\int x\, dm}{\int dm} \qquad dm = \rho\, dx$$

y

$$dm = a\left(1 + \frac{x^2}{L^2}\right) dx$$



$$\int dm$$

$$dm = a\left(1+\frac{x^2}{L^2}\right)dx$$

$$M = \int dm = \int_0^L a\left(1+\frac{x^2}{L^2}\right)dx$$

$$= a\int_0^L 1+\frac{x^2}{L^2}\,dx$$

$$= a\left(L+\frac{L^{\cancel{3}}}{3\cancel{L^2}}\right)$$

$$= a\left(L+\frac{L}{3}\right)$$

$$a\left(\frac{4}{3}L\right)$$

$$\int x\,dm = \int_0^L x\left(a\left(1+\frac{x^2}{L^2}\right)\right)dx$$

$$= a\int_0^L x+\frac{x^3}{L^2}\,dx$$

$$= a\left(\frac{L^2}{2}+\frac{L^{\cancel{4}}}{4\cancel{L^2}}\right)$$

$$= a\left(\frac{L^2}{2}+\frac{L^2}{4}\right)$$

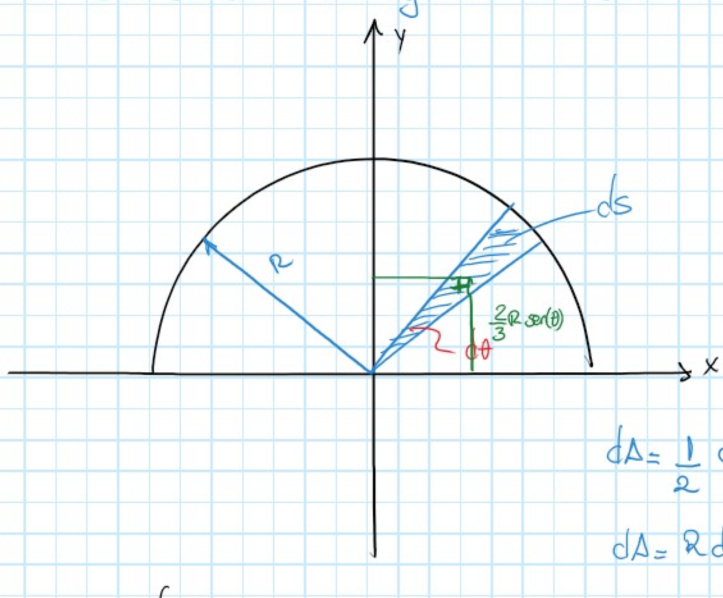
$$= a\left(\frac{3}{4}L^2\right)$$

$$\frac{9}{16}L$$

$$\frac{\cancel{a}\,\frac{3}{\cancel{4}}\,L^{\cancel{2}}}{\cancel{a}\,\frac{4}{3}\cancel{L}}$$

$$\frac{9}{16}L$$

Placa semicircular homogenea de radio R



$$dA = \frac{1}{2}\,ds\,R$$

$$dA = R\,ds\,\frac{1}{2} = \frac{1}{2}R.\;R\,d\theta$$

$$= \frac{1}{2}R^2 d\theta =$$

$$y_{CM} = \frac{\int y\,dA}{\int dA}$$

$$y_{CM} = \frac{\int y\, dA}{\int dA}$$

$$\int y\, dA = \int \frac{\cancel{2}}{3} R \operatorname{sen}(\theta) \frac{1}{\cancel{2}} R^2 d\theta$$

$$= \frac{R^3}{3} \int_0^{\theta} \operatorname{sen}(\theta)\, d\theta$$

$$= \frac{R^3}{3} (-\cos(\theta)) \Big|_0^{\theta}$$

$$= \frac{R^3}{3} (1 - \cos(\theta))$$

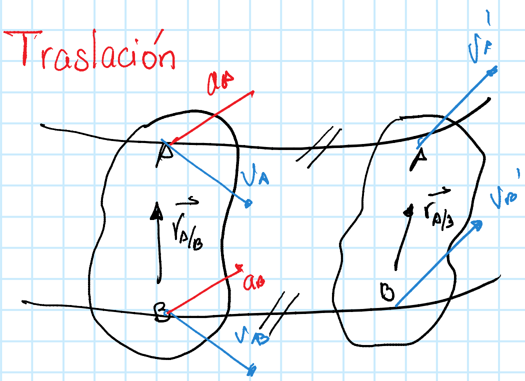
$$= \frac{1}{2} R^2 d\theta =$$

$$A = \int \frac{1}{2} R^2 d\theta = \frac{1}{2} R^2 \theta \;//$$

$$y_{CM} = \frac{\frac{R^3}{3} (1 - \cos(\theta))}{\frac{1}{2} R^2 \theta}$$

$$\frac{2}{3} \frac{R (1 - \cos(\theta))}{\theta}$$

Traslación



$$\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B$$

$$\frac{d[\vec{r}_{A/B}]}{dt} = \frac{d\vec{r}_A}{dt} + \frac{d\vec{r}_B}{dt}$$

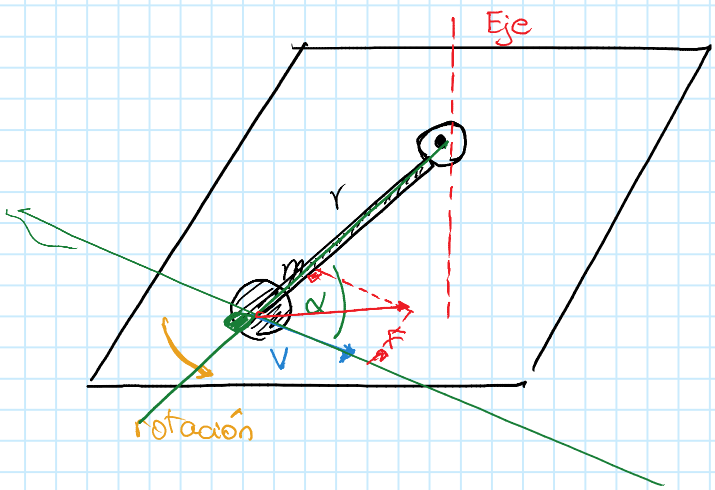
$$\vec{V}_{A/B} = \vec{V}_A + \vec{V}_B = 0$$

$$\vec{V}_A = \vec{V}_B$$

$$\vec{a}_A = \vec{a}_B = \text{centro de masa}$$

Rotación alrededor de un eje fijo:

superficie horizontal lisa



$\vec{F}$ = fuerza horizontal

$F = cte$, $\alpha = cte$

$$\sum F_x = m a_x$$

$$F \operatorname{sen}(\alpha) = m a_T$$

$$F \operatorname{sen}(\alpha) = m \alpha r$$

$$\frac{r}{r} \quad \frac{F \operatorname{sen}(\alpha)}{m r} = \alpha$$

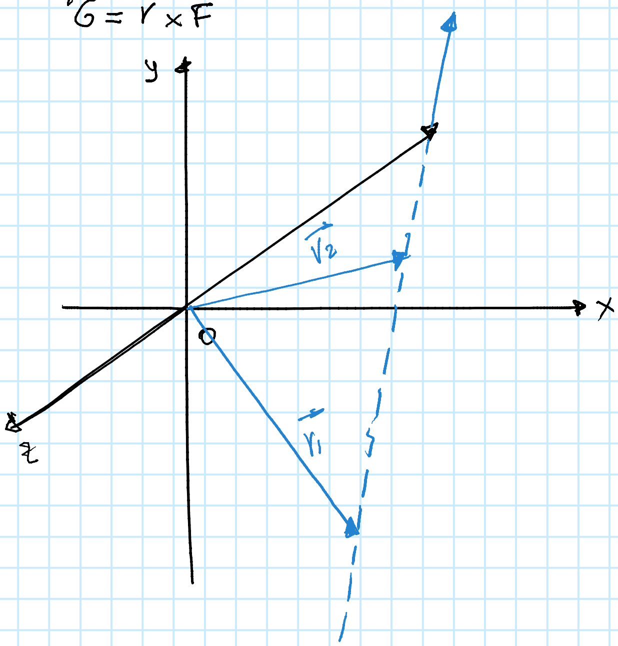
$$\alpha = \frac{r F \operatorname{sen}(\alpha)}{m r^2}$$

Torque (Momento) $\vec{\tau}$

Se calcula alrededor de un centro de giro

Efecto rotativo de una fuerza que actua sobre n cuerpo en función de un radio de giro.

$\vec{\tau} = \vec{r} \times \vec{F}$



→ magnitud $|\vec{\tau}| = \tau = |\vec{r} \times \vec{F}| = rF\,sen(\theta)$

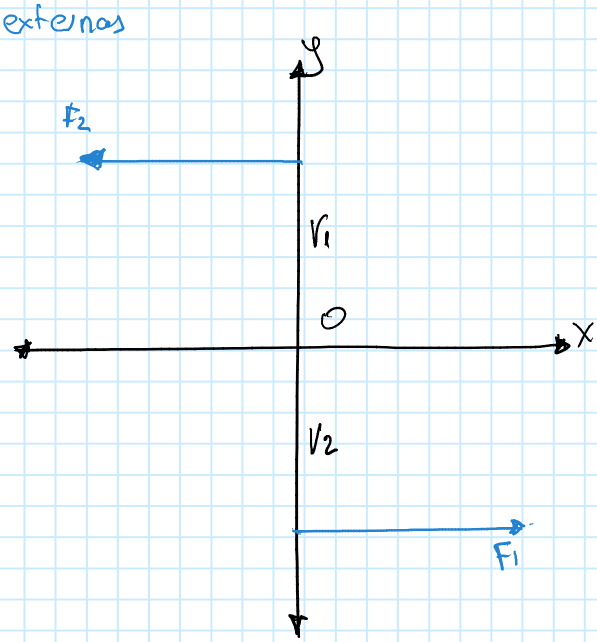
→ dirección $\vec{\tau} \perp \vec{r} \wedge \vec{F}$

regla de la mano derecha

$0° < \theta < 180°$, mN , Nm

Torque externo neto / Torque resultante : $\sum \vec{\tau}$

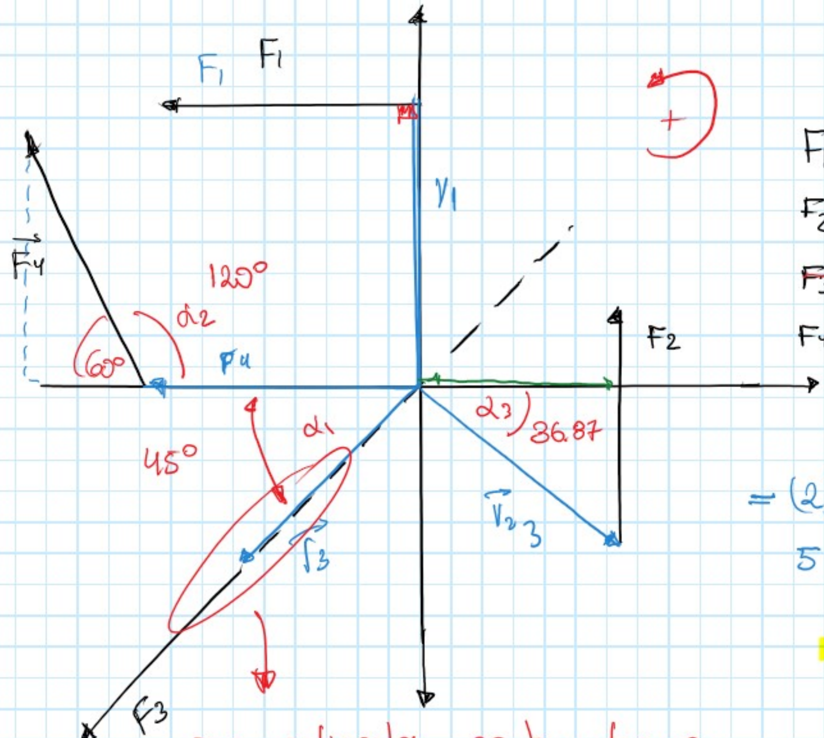
↳ Fuezas externas



$F = F_1 = F_2$

$r = r_1 = r_2$

$$\sum \vec{\tau}_o = \vec{\tau}_1 + \vec{\tau}_2$$
$$= r(F_1)\vec{k} + r(F_1)\vec{k}$$
$$= 2rF\vec{k}$$



$F_1 = 5$ N $\quad r_1 = 2$

$F_2 = 10$ N $\quad r_2 = 3$

~~$F_3 = 15$ N $\quad r_3 = 4$~~

$F_4 = 20$ N $\quad r_4 = 5$

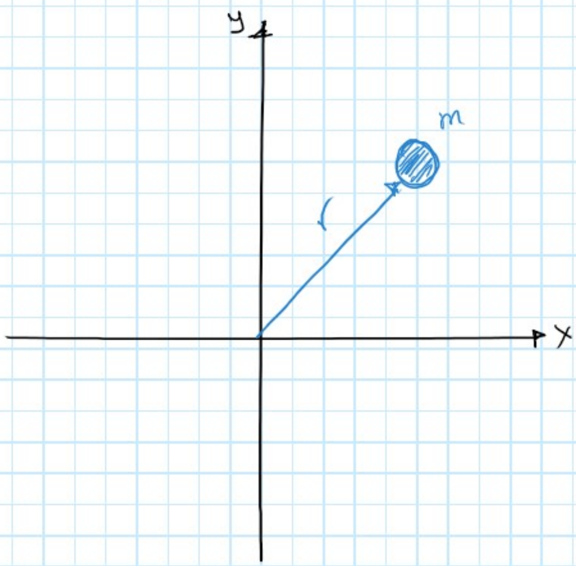
$= (2)(5)\,\text{sen}(90°) + 3\cos(36.87)\,10 +$
$5\,(20\,\text{sen}\,(60°))$

$= -52.6025$

son colineales no hay torque

$$\alpha = \frac{\sum \vec{\tau}}{mr^2}$$

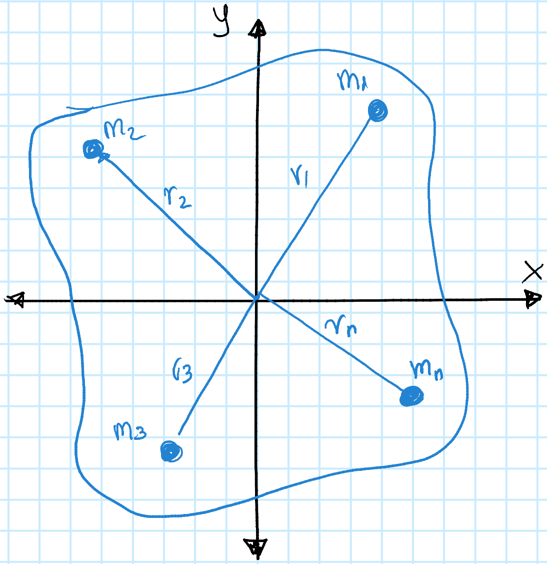
Momento de inercia o inercia rotacional: (I)



Alrededor de O

$I = mr^2 \quad , \; kg\,m^2$

sistema n partículas



Alrededor de O

$$I = I_1 + I_2 + \dots + I_n$$

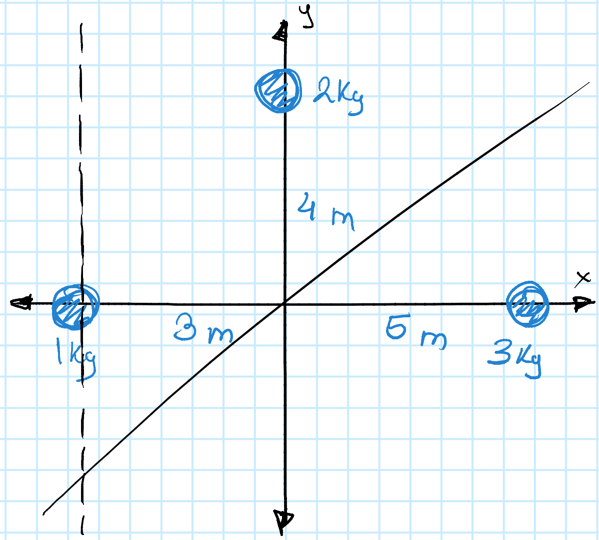
$$= m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2$$

$$= \sum_{i=1}^{n} m_i r_i^2$$

Alrededor del eje x

$$I_x = I_{1x} + I_{2x} + \dots I_{nx}$$

$$= \sum_{i=1}^{n} m_{ix} r_{ix}^2$$



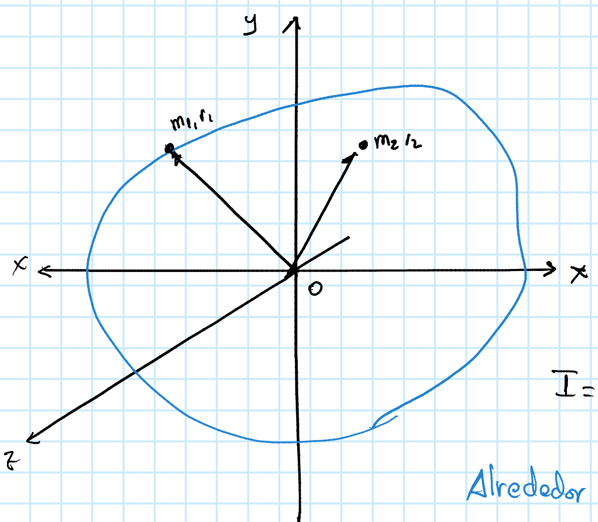
$I_O = 116 \ kg\, m^2$

$I_x = 32 \ kg\, m^2$

$I_y = 84 \ kg\, m^2$

$I_1 = 210 \ kg\, m^2$

$I_z = 242 \ kg\, m^2$

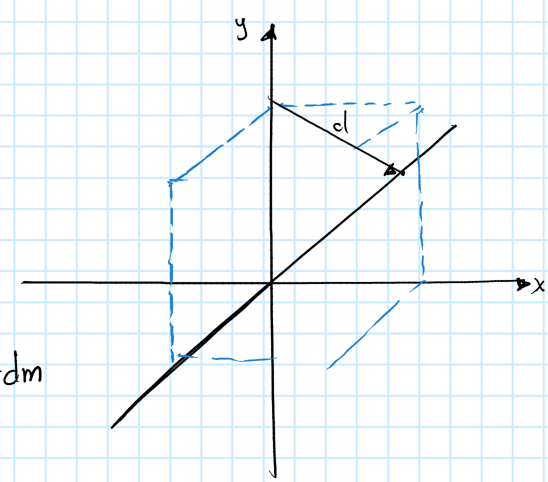


$$I = \sum_{i=1}^{n \to \infty} \underset{\downarrow 0}{m} r^2$$

$$I = \int r^2 \, dm$$

$$I = \int (x^2 + y^2 + z^2) \, dm$$

$$I = \int x^2 dm + \int y^2 dm + \int z^2 dm$$



Alrededor de y

$$I_y = \int d^2 \, dm = \int (x^2 + z^2) \, dm$$

$$I_y = \int (x^2 + z^2)\, dm$$

$$I_x = \int (y^2 + z^2)\, dm$$

$$I_z = \int (x^2 + y^2)\, dm$$

$$J = I_x + I_y + I_z$$

$$= \int 2(x^2 + y^2 + z^2)\, dm$$

$$= 2\int (x^2 + y^2 + z^2)\, dm$$

$$= 2 I_O$$

$$I = \int r^2\, dm$$

$$I = C\, M L^2$$

C coeficiente adimensional

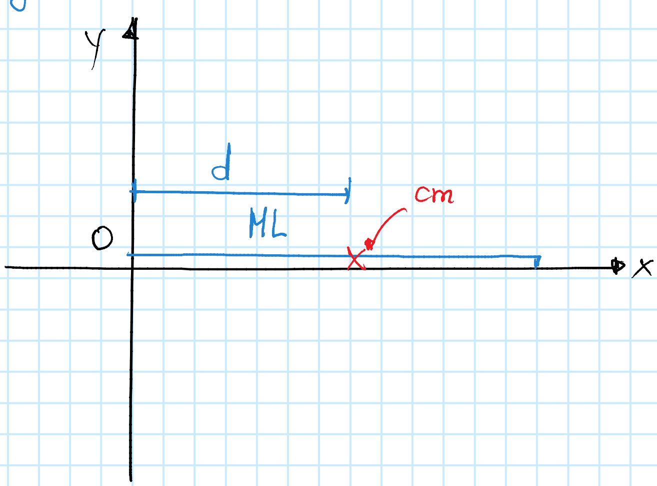
$$I = k^2 M$$

$$k = \sqrt{\frac{I}{M}} \text{, radio de giro}$$

Varilla homogenea

masa: M

longitud: L



$$\rho = \frac{dm}{dL} = cte$$

$$I = \int r^2\, dm$$

$$I = \int x^2\, dm$$

$$I = \int x^2 \rho\, dx$$

$$I = \rho \int_0^L x^2\, dx$$

$$I = \rho \frac{L^3}{3}$$

$$I = \rho L^2 \cdot \frac{L}{3}$$

$$I = \int_{-L/2}^{L/2} x^2 \rho\, dx$$

$$I = \rho \int^{L/2} \ldots$$

$$I = \rho \int_{-L/2}^{L/2} x^2 \, dx$$

$$I = \rho \, \frac{x^3}{3} \Big|_{-\frac{L}{2}}^{L/2}$$

$$I = \rho \, \frac{L^3}{12}$$

$$I = \rho \cdot L \cdot L^2 \, \frac{1}{12}$$

$$I = M L^2 \, \frac{1}{12}$$

$$\boxed{I = \frac{1}{12} M L^2}$$

$$I = \rho L^2 \cdot \frac{L}{3}$$

$$\boxed{I = \frac{1}{3} M L^2}$$

Teorema de los ejes paralelos o método de Steiner

$$I = I_{CM} + M d^2$$

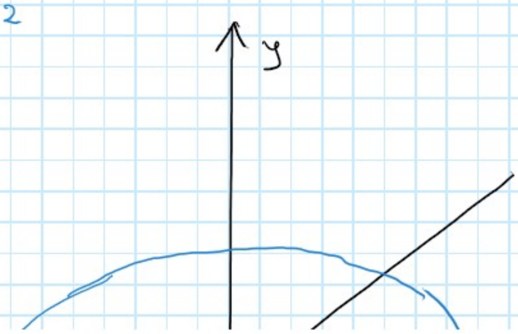
$$= \frac{1}{3} ML^2 + M\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{3} ML^2 + \frac{M L^2}{4}$$

$$= \frac{4ML^2 - 3ML^2}{12}$$

$$= ML^2 \, \frac{1}{12}$$

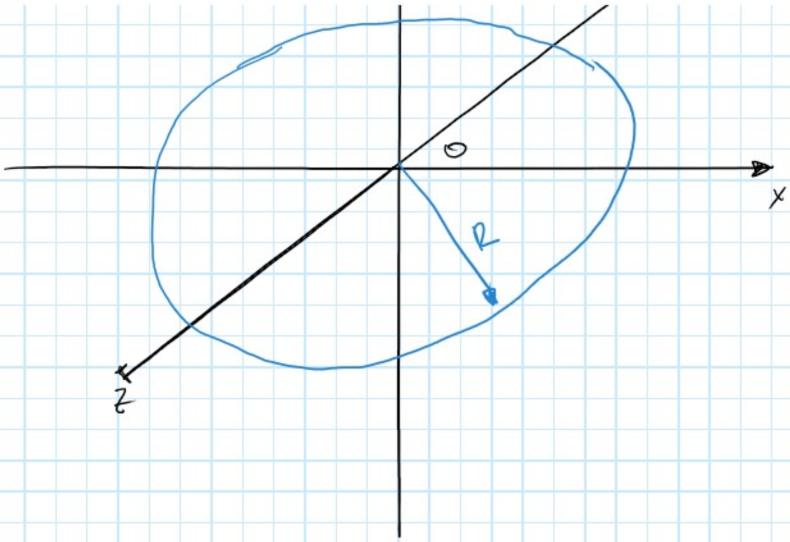
$$= \frac{1}{12} M L^2$$



Alrededor de O

$$\int r^2 \, dm \qquad \rho = \frac{dm}{dS}$$

$$dm = \rho \, dA$$



$dm = \rho\, dA$

$dA = 2\pi r\, dr\, \rho$

$$I_0 = \int r^2 \rho (2\pi r\, dr)$$

$$= \rho 2\pi \int_0^r r^3\, dr$$

$$= \rho 2\pi \left.\frac{R^4}{4}\right|_0^R$$

$$= \rho 2\pi \frac{R^4}{4}$$

$$= \rho \frac{\pi R^2}{2} \cdot R^2$$

$$= \frac{1}{2} M R^2$$

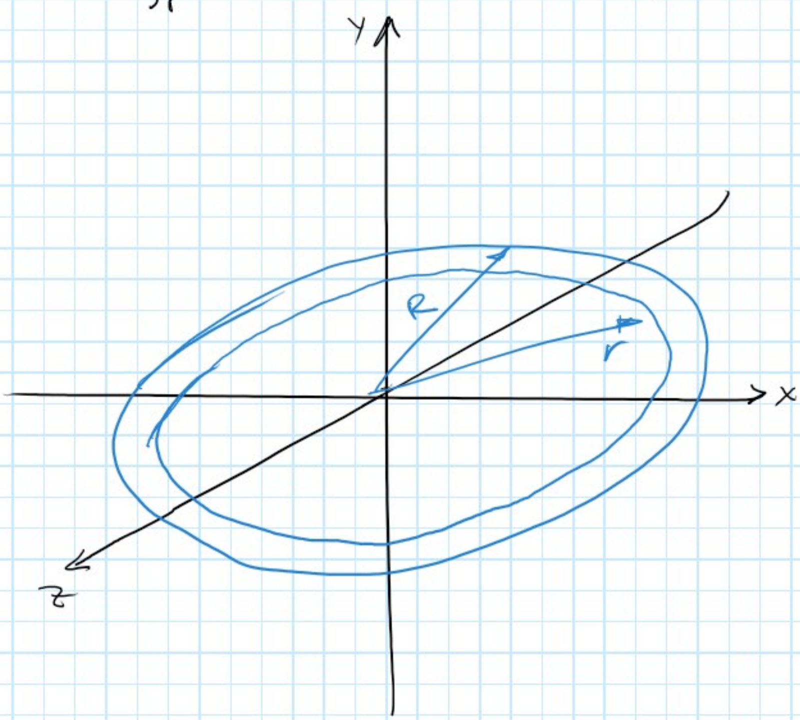
$$I_o = I_x + I_y$$

$$\frac{1}{2} M R^2 = 2 I_x$$

$$I_x = \frac{1}{4} M R^2$$

Momento de Inercia de un anillo

$$I_o = \int_r^R r^2\, dm \qquad dm = \rho\, dA$$



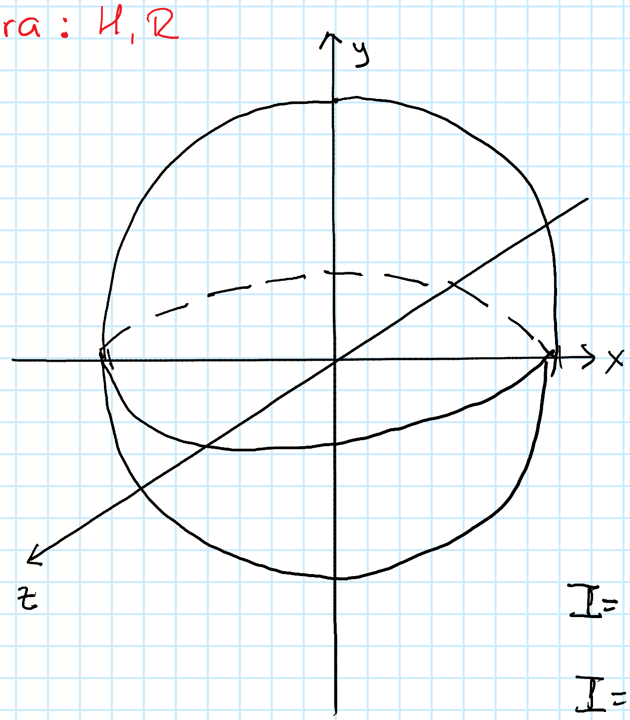
$M \quad R_1, R_2$

$\frac{1}{2} M (R_1^2 + R_2^2)$

Momento de Inercia

Thursday, January 24, 2019 4:06 PM

Esfera : M, R



momento de inercia

$$I = \int r^2 dm$$

$$V = \int_0^R 4\pi r^2 dr$$

$$V = 4\pi \frac{r^3}{3}\Big|_0^R \qquad V = \frac{4\pi R^3}{3}$$

$$dV = 4\pi r^2 \, dr \qquad \rho = \frac{dm}{dV}$$

$$dm = \rho \, dV$$

$$I = \int r^2 dm$$

$$I = \int r^2 \rho \, 4\pi r^2 dv$$

$$I = \int r^4 4\pi \rho \, dv$$

$$I = 4\rho\pi \int r^4 \, dr$$

$$I = 4\pi\rho \, \frac{r^5}{5}\Big|_0^R$$

$$I = 4\pi\rho R^3 \, \frac{R^2}{5} \cdot \frac{3}{3}$$

$$I = \frac{4\pi R^3}{3} \cdot \frac{R^2}{5} \rho$$

$$I = M \cdot 3 \, \frac{R^2}{5}$$

$$I = \frac{3}{5} M R^2 \; //$$

$$2 I_0 = I_x + I_y + I_z$$

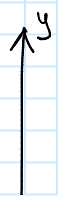
$$2 I_0 = 3 I$$

$$2 \, \frac{\cancel{3}}{5} I_0 = \cancel{3} I$$

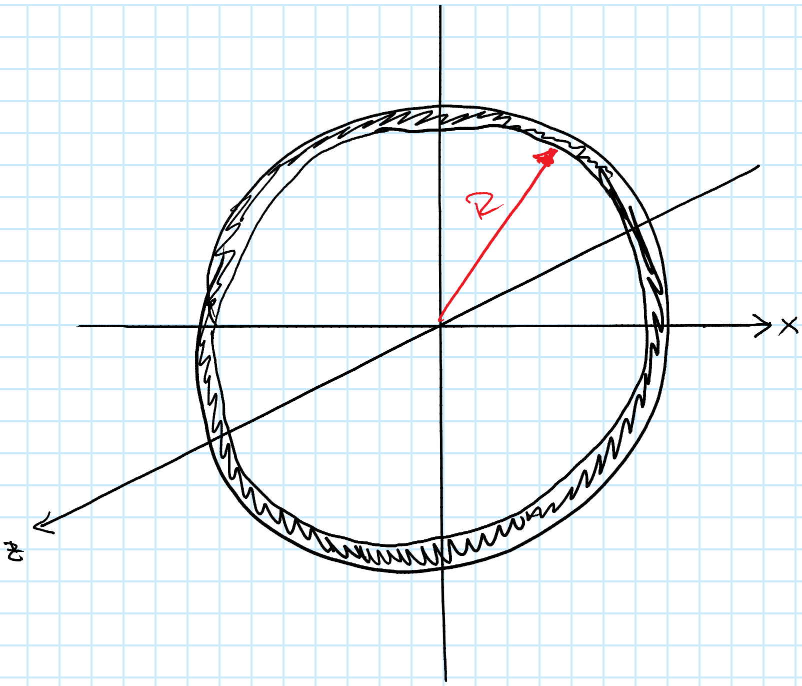
$$\frac{2}{5} I_0 = I$$

$$I = \frac{2}{5} M R^2$$

Esfera Hueca M.R.



$$I = \int_0^R r^2 dm$$



$$I = \int_0^R r^2 \, dm$$

$$I = R^2 \int dm$$

$$I_0 = MR^2$$

$$2 I_0 = 3 I_{x,y,z}$$

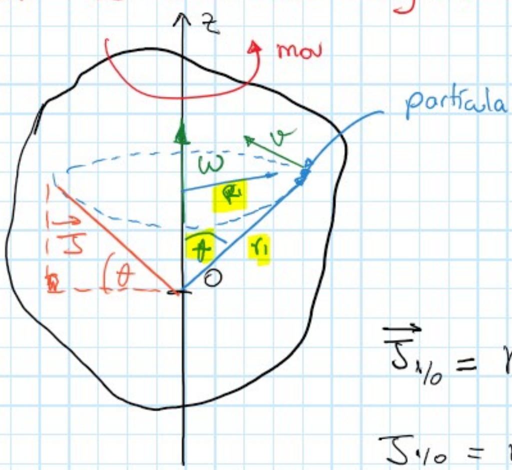
$$2 I_0 = 3 MR^3$$

$$I_0 = \frac{3}{2} MR^2$$

Cantidad de movimiento angular

Thursday, January 24, 2019 4:31 PM

Cantidad de movimiento angular de un cuerpo rígido ($\vec{J}, \vec{L}, \vec{H}$)



$\vec{V} = \vec{\omega} \times \vec{r_1}$ $\vec{V} \perp \vec{\omega} \times \vec{r_1}$

$V_1 = \omega r_1 \operatorname{sen}(\theta)$

$V_1 = \omega R_1$

$\vec{J}_{1/O} = m_1(\vec{r_1} \times \vec{V_1})$

$J_{1/O} = m_1 r_1 v_1$

$\vec{J}_{1/z} = m_1 r_1 v_1 \operatorname{sen}(\theta_1)$

$\vec{J}_{1/z} = m_1 V_1 R_1$

$= mR^2 \omega$

momento de inercia

$J_{s/z} = m_1 R_1^2 \omega_1 + m_2 R_2^2 \omega_2 + \dots$

$= \left(\sum_{i=1}^{n} m_i R_i^2\right) \omega$

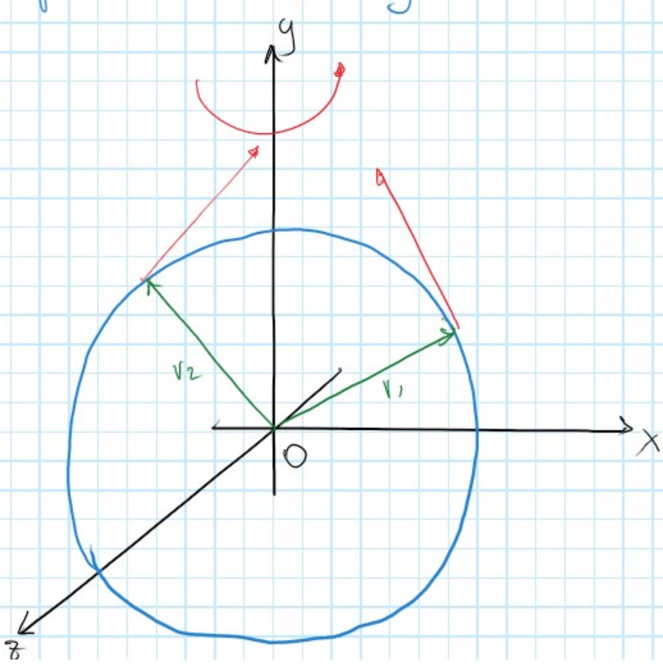
$= I_z \omega$

$\vec{J} = \vec{m_i}(\vec{r_i} \times \vec{v_i})$

$J_s = \vec{J_1} + \vec{J_2} + \vec{J_3} + \vec{J_4} \dots$

Cantidad de movimiento no siempre es paralela a la velocidad angular.

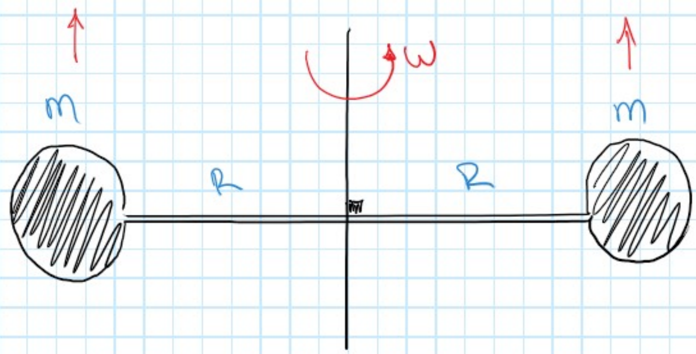
solo son paralelos cuando gira sobre uno de los 3 ejes principales



Sobre un eje princial la cantidad de movimiento angular

$\vec{J} = I \vec{\omega}$

al menos 3 determinan la simetría.

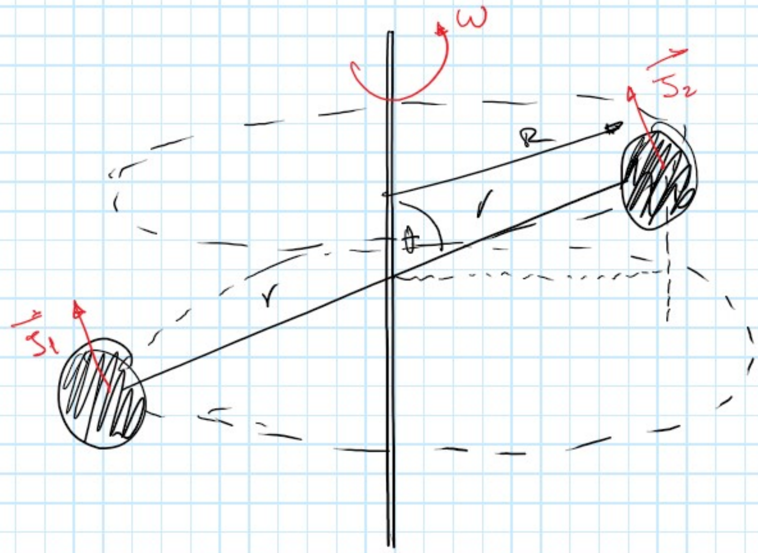


$\vec{J}_s = \vec{J}_1 + \vec{J}_2$

$\vec{J}_s = mr^2\omega + mr^2\omega$

$J_s = 2mr^2(\omega)$

$J_s = I\,\omega$ del sistema



$J = mrv\,\mathrm{sen}(90°)$

$J = mrv$

$v = \omega R$

$J = mr\omega R$

$J = \dfrac{mR^2\omega}{\mathrm{sen}(\theta)}$

$J = \dfrac{mR^2\omega}{\cancel{\mathrm{sen}(\theta)}} \cdot \cancel{\mathrm{sen}(\theta)} = mR^2\omega$

Ecuación del movimiento rotacional de un cuerpo rígido

S: n partículas

$$\sum_i \vec{\tau} = \frac{d\vec{J}}{dt} \quad , \quad \sum_i \vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \dots$$

$$\vec{J} = \vec{J}_1 + \vec{J}_2 + \vec{J}_3 + \dots$$

Para un cuerpo rígido

$$\vec{J} = I\vec{\omega}$$

$I \rightarrow$ cte

$$\sum_i \vec{\tau} = \frac{d(I\vec{\omega})}{dt}$$

$$\sum_i \vec{\tau} = I\frac{d(\vec{\omega})}{dt}$$

$$\sum_i \vec{\tau} = I\vec{\alpha} \quad (2^{da} \text{ Ley de Newton})$$

Ecu. Instantanea y Vectorial

Equilibrio Rotacional

Para cuerpo rígido

$$\sum_i \vec{\tau} = I\vec{\alpha}$$

Si $\vec{\omega}$ $= \vec{0}$, no rota

$\neq \vec{0}$, rota, gira uniformemente

$$\Delta\vec{\omega} = \vec{0} = \vec{\alpha}$$

$$\sum_i \vec{\tau} = \vec{0} \text{ , ER}$$

$\sum \vec{F} = \vec{0}$ E.T

$\sum_i \vec{\tau} = \vec{0}$ E.R

Equilibrio → Reposo
→ Movimiento

$$\sum_i \vec{\tau} = \frac{d\vec{J}}{dt}$$

$$dt \sum_i \vec{\tau} = d\vec{J}$$

$$\underbrace{\int_{t_0}^{t_f} \sum \vec{\tau}\, dt}_{I_{A\,NETO}} = \int_{J_0}^{J_f} d\vec{J}$$

$$I_{A\,NETO} = \Delta\vec{J}$$

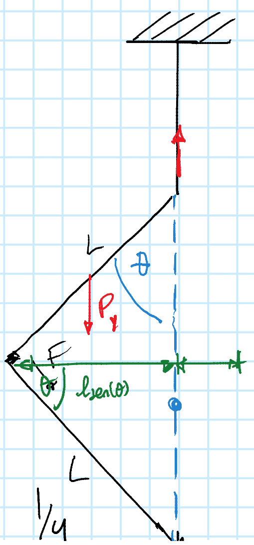
Si $\sum \vec{\tau} = \vec{0}$, $I_{A\,NETO} = \vec{0}$

$\Delta\vec{J} = 0$

$\vec{J_f} = \vec{J_0}$

$\vec{J} = cte$ Principio de conservación de la cantidad de movimiento angular

sistemas aislados de torque



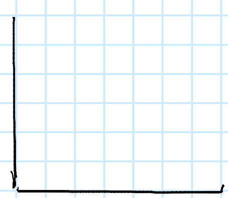
$\sum \vec{\tau} = 0$

$$sen(\theta) = \frac{\frac{L}{4}}{\frac{L}{1}}$$

$$sen(\theta) = \frac{1}{4}$$

$$\theta = sen^{-1}\left(\frac{1}{4}\right)$$

$$\theta = 14.4775$$

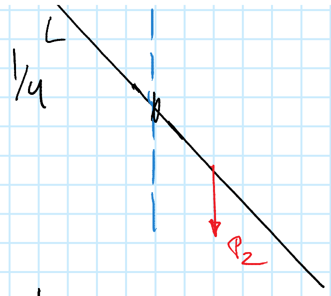


$\sum \vec{F} = \vec{0}$ $\sum \vec{\tau} = \vec{0}$

$\sum \vec{\tau} = 0 +$

$$+P_1\frac{l}{2} sen(\theta) + \left(-P_2\left(\frac{l}{2}\cos(\theta) - l\,sen(\theta)\right)\right) = 0$$

$$\frac{sen(\theta)}{2} + sen(\theta) - \frac{\cos(\theta)}{2} = 0$$



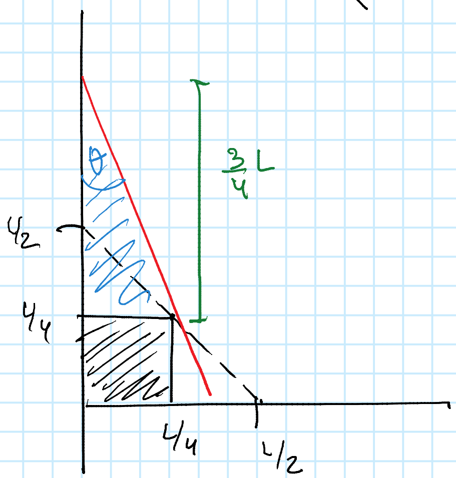
$\theta = 14.4775$

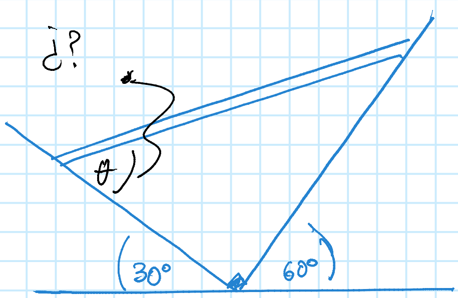
$$\frac{sen(\theta)}{2} + sen(\theta) - \frac{\cos(\theta)}{2} = 0$$

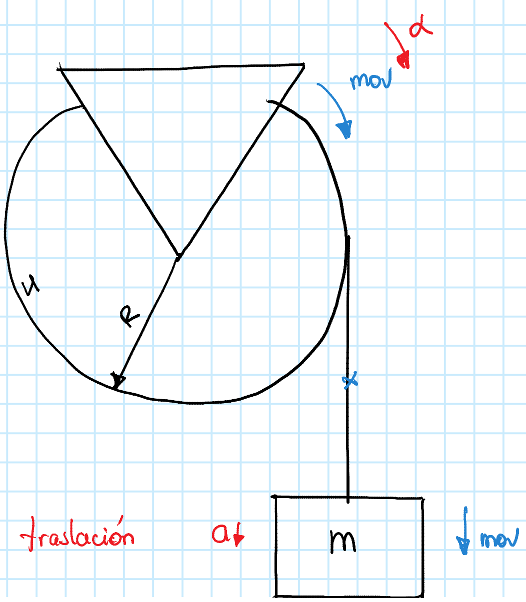
$$\frac{3}{\cancel{2}} sen(\theta) = \frac{1}{\cancel{2}} \cos(\theta)$$

$$\tan(\theta) = \frac{1}{3}$$

$$\theta = 18.4349°$$

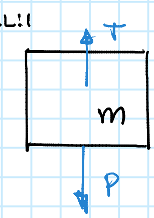




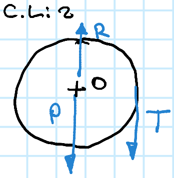


$a = |\vec{a}_T| = \alpha R$

D.C.L: 1



D.C.L: 2



$$\sum F_y = m a_y$$

$$P - T = m a_y$$

$$mg - T = m a_y \qquad (1)$$

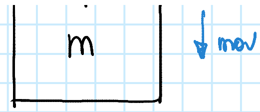
$$\sum \vec{\tau} = I \vec{\alpha}$$

$$\sum \vec{\tau} = I \vec{\alpha}$$

$$TR = \frac{1}{2} M R^2 \alpha$$

$$T = \frac{1}{2} M a \qquad (2)$$

traslación $a\downarrow$



$T = \frac{1}{2} M a$ (2)

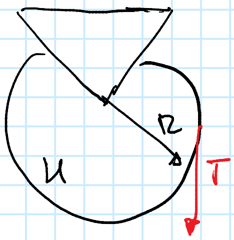
$mg - \frac{1}{2} M a = m a$

$mg = ma + \frac{1}{2} M a$

$mg = a\left(m + \frac{1}{2} M\right)$

$\frac{mg}{m + \frac{1}{2} M} = a$

$\alpha = \frac{mg}{R\left(m + \frac{1}{2} M\right)}$

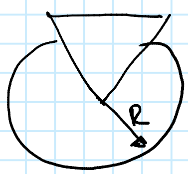


$\sum_i \vec{\tau} = I\alpha$

$TR = \frac{1}{2} M R^2 \alpha$

$T = \frac{1}{2} M a$

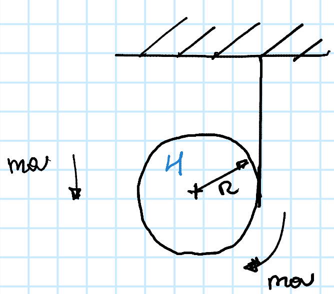
$\alpha = \frac{mg}{\frac{1}{2} M R}$



$\sum_i \vec{\tau} = \vec{I}\alpha$

$mgR = \left(\frac{1}{2} M R^2 + m R^2\right)\alpha$

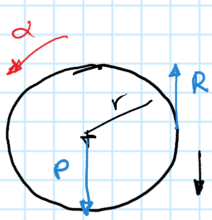
$\alpha = \left(\frac{mg}{\frac{1}{2} M R + m R}\right)$



$\alpha = ¿?$

$a_{cm} = ¿?$

DCL



$\sum_i F = ma$

$P - T = ma$

$mg - T = ma$

$mg - \frac{1}{2} ma = ma$

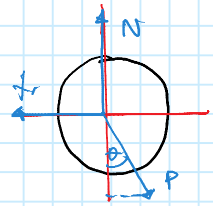
$\sum_i \tau = I\vec{\alpha}$

$R(r) = \left(\frac{1}{2} M R^2\right)\alpha$

$T = \frac{1}{2} M a$

$$\cancel{m}g = \frac{3}{2}\cancel{m}a$$

$$a = \frac{2g}{3}$$

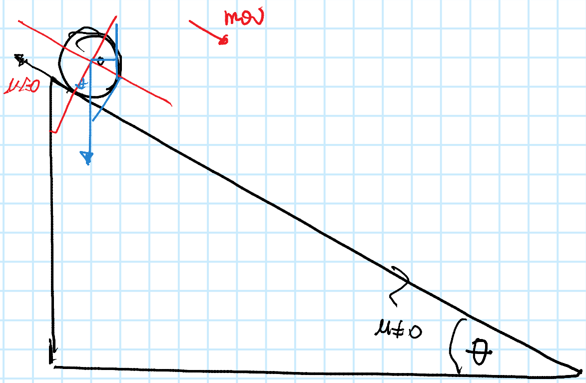


$$P_x = P\,\text{sen}(\theta)$$
$$P_y = P\cos(\theta)$$

$$\sum \tau = I\alpha$$

$$\cancel{R} f_r = \frac{2}{5} \cancel{R} M a_x$$

$$f_r = \frac{2}{5} M a_x$$



$$\sum F_y = 0$$
$$N = P\cos(\theta)$$

$$\sum F_x = m a_x$$
$$P_x - f_r = m a_x$$
$$mg\,\text{sen}(\theta) - \frac{2}{5} M a = m a$$
$$\cancel{m} g\,\text{sen}(\theta) = \frac{7}{5}\cancel{m} a$$
$$a = \frac{5\,g\,\text{sen}(\theta)}{7}$$

Energia Cinética Rotacional

S: n partículas

$$E_c = \frac{1}{2}\sum_{i=1}^{n} m_i v_i^2 \qquad V_i = \omega R_i$$

$$E_c = \frac{1}{2}\left(\sum_{i=1}^{n} m_i R_i^2\right)\omega^2$$

$$E_{c_{rot}} = \frac{1}{2} I \omega^2$$

$$E_c = E_{c\ldots} + \frac{1}{2} M V_{c}^2$$

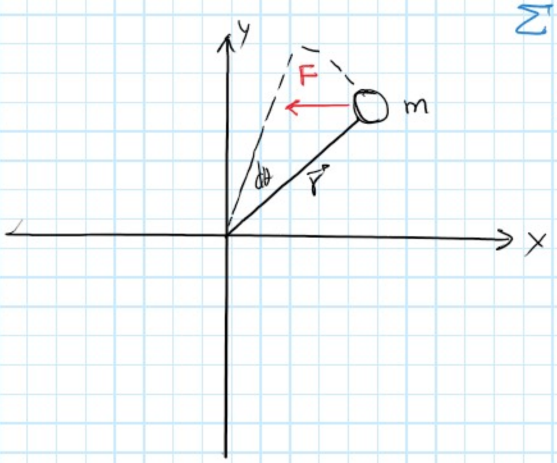
$$Ec_{rot} = \frac{1}{2} I \omega^2$$

$$Ec = Ec_{/CM} + \frac{1}{2} M V_{CM}^2$$

Ec traslación

Ec/rotación

$$Ec = \frac{1}{2} I\omega^2 + \frac{1}{2} M V_{CM}^2$$



$$\sum \vec{\tau} = I\alpha$$

$$\sum \tau = I \frac{d\omega}{dt} \cdot \frac{d\theta}{d\theta}$$

$$\underbrace{\int_{\theta_0}^{\theta_f} \sum \vec{\tau}\, d\theta}_{} = \int_{\omega_0}^{\omega_f} I\, \omega\, d\theta$$

$$T_\tau = I \frac{\omega^2}{2} \Big|_{\omega_0}^{\omega_f}$$

$$T_\tau = \Delta Ec_{rotacional}$$

$$\text{Potencia} = \frac{dT}{dt}$$

$$\frac{\tau\, d\theta}{dt}$$

$$\tau\, \omega$$

$$\sum_i F_c + \sum_i F_{NC} = \Delta Ec$$

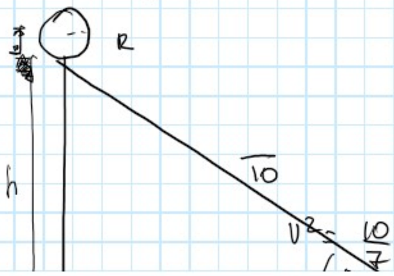
$$\Delta E_p \qquad \sum T_{ext} \qquad (-Q)$$

$$\sum_i T_{ext} = \Delta Ec + \Delta E_p + Q$$

$$\sum_i T_{ext} \qquad \Delta Ec_T$$

$$+ \qquad +$$

$$\sum_i F_{Gret} \qquad \Delta Ec_{ROT}$$



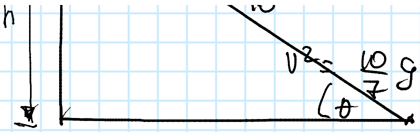
$$v^2 = \frac{10}{7} g ($$

$$\Delta E_M = 0$$

$$E_{Mf} = E_{Mo}$$

$$Ec_T + Ec_R = E_{pg}$$

$$= mgh$$



$v^2 = \frac{10}{7} g ($ $\qquad = mgh$

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = mg(h+R)$$

$$\frac{1}{2}\cancel{M}v^2 + \frac{1}{2}\,\frac{2}{5}\cancel{M}R^2\omega^2 = \cancel{m}g(h+R)$$

$$\frac{1}{2}v^2 + \frac{1}{5}v^2 = g(h+R)$$

$$7v^2 = g(h+R)$$

$h+R)$

Movimiento Armónico Simple

Monday, February 4, 2019 4:04 PM

Movimiento oscilatorio

La trayectoria debe ser sobre la misma linea

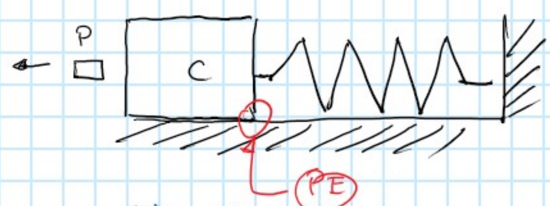
- libres
- forzadas
- amortiguadas

ejemplo 1

$M = 2500 \text{ Kg}$

$m = 100 \text{ Kg}$

$v_{q/c} = -150 \text{ m/s}$



S: P - C durante el disparo

$\Delta\vec{p} = 0$

$P_f = P_o$

$m_p v_p^{0} + m_c v_c^{0} = m_p v_p' + m_c v_p'$

$m_p v_p' = -m_c v_c'$

$m_p (150 \text{ m/s} + V_c) = -m_c V_c'$

$(100)(-150 + V_c) = -2500 V_c$

$V_c = +5.7692 \text{ m/s}$

$\vec{V}_{p/c} = 150 \text{ m/s} = \vec{V}_p - \vec{V}_c = 150 \text{ m/s}$

$\vec{V}_p = 150 \text{ m/s} + \vec{V}_c$

MAS:

$x = f(t)$

$= C_1 \text{ sen}(\omega t) + C_2 \cos(\omega t)$

$t_0 = 0 s$, $X_0 = 0$ $V_0 =$

$V_{x_0} = 5.77 \text{ [m/s]}$

$\omega_n = \sqrt{\frac{k}{m}} = 6.66 \text{ (rad/s)}$

frecuencia natural

$C_1 = \frac{V_{x_0}}{\omega_n}$

$C_2 = 0$

$x = 0.866 \text{ sen}(6.66 t)$

$V_x = 5.77 \cos(6.66 t)$

$a_x = -38.42 \text{ sen}(6.66 t)$

$V_x = 0$

$0 = \cos(6.66 t)$

$\cos^{-1}(0) = 6.66 t$

$\frac{\pi}{2} = 6.66 t$

$t = 0.2358 \text{ [s]}$

$x_{t=0.2358} = 0.02373$

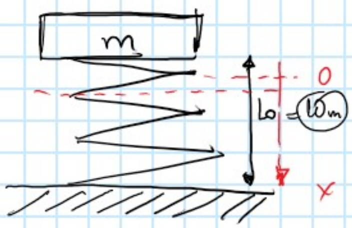
Ejercicio 2

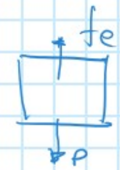
Ejercicio 2

Un cuerpo cuya masa es $m = 18\,kg$ se abandona desde el reposo en el extremo superior libre de un resorte $k = 330\,N/m$ / $t = 0.3\,[s]$

¿Compresión?

$$\omega_n = \sqrt{\frac{k}{m}} = 4.2817 \quad rad/s$$





$\Sigma F_x = 0$

$P = fe$

$mg = kx$

$x = \frac{mg}{k}$

$x = 0.5345$

$$x = C_1 \operatorname{sen}(\omega_n t) + C_2 \cos(\omega_n t)$$

$t_0 = 0\,s \qquad V_{x_0} = 0 \qquad X_0 = -0.5345\,[m]$

$\Downarrow$

C_2

$$x = -0.5345 \cos(4.2817\,t)$$

$$x = -0.151199\,[m]$$

$$\Delta x = 0.3833\,[m]$$

Energía en el movimiento armónico simple.

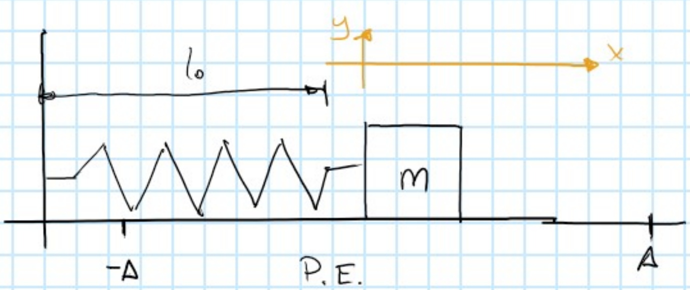
Consideraciones { superficie horizontal lisa

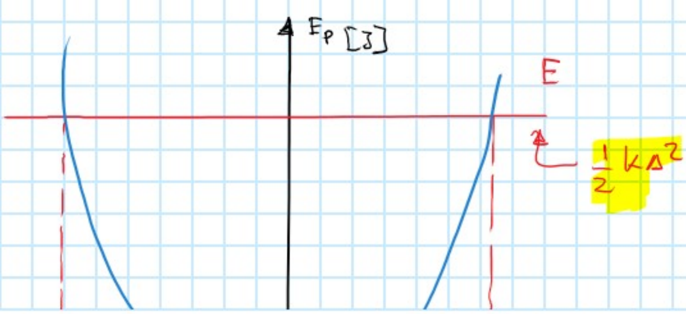
S: masa - resorte.

$\Delta E_M = 0$

$E_M = E_c + E_p = cte$

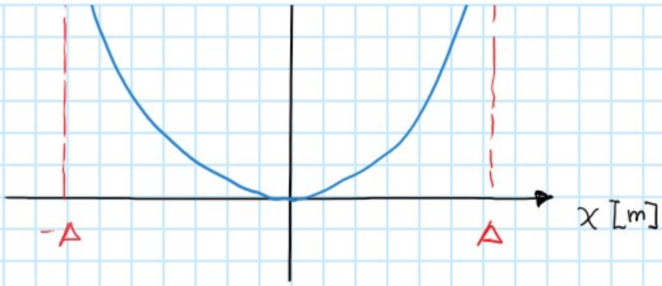
$$\frac{1}{2} m v^2 + \frac{1}{2} k x^2 = cte$$





$$\frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$$

$$m v^2 + k x^2 = k A^2$$



$$mv^2 + kx^2 = kA^2$$

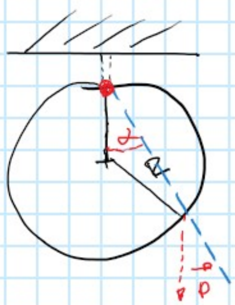
$$v^2 + \omega^2 x^2 = \omega^2 A^2$$

$$v^2 = \omega_n^2 (A^2 - x^2)$$

$$v = \pm \omega_n \sqrt{A^2 - x^2}$$

En vertical la posición no coincide con la deformación

Péndulo Físico (Cuerpo rígido ≠ péndulo simple)



$$\sum \vec{\tau} = I\vec{\alpha}$$

$$\sum \tau_o = I_o \alpha \quad +\circlearrowleft$$

$$-PR\, sen(\theta) = \frac{3}{2} MR^2 \alpha$$

$$-\cancel{M} g \cancel{R}\, sen(\theta) = \frac{3}{2} \cancel{M} R^{\cancel{2}} \alpha \qquad \theta \approx sen(\theta)$$

$$\alpha = -\frac{2}{3} \frac{g\, sen(\theta)}{R}$$

$$\alpha \cong -\frac{2}{3} \frac{g}{R} \theta$$

$$\ddot{\theta} \cong -\frac{2}{3} \frac{g}{R} \theta$$

$$\ddot{\theta} + \frac{2}{3} \frac{g}{R} \theta = 0$$

q = posición del sistema

$$\omega_n = \sqrt{\frac{2}{3} \frac{g}{R}} \quad \leftarrow \text{coeficiente constante}$$

$$\ddot{q} + \omega_n^2 q = 0$$

siempre positiva

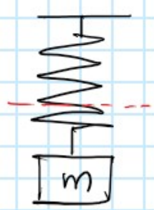
Ejercicio 3

Datos

$m = 300g$

$x_e = 20$ cm.

$A = 5$ cm.



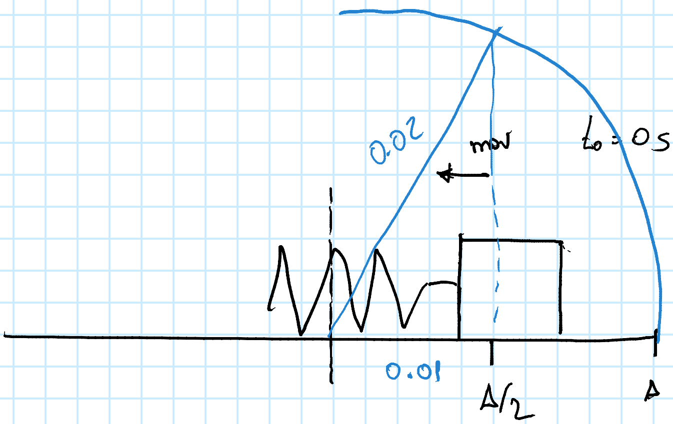
$$P = kx \qquad \omega_n = \sqrt{\frac{k}{m}} = 7 \text{ rad/s}$$

$$mg = kx$$

$$k = \frac{mg}{x} = 14.7$$

$$T = \frac{2\pi}{\omega_n} = 0.8975 \text{ [s]} \qquad V_{máx} = A\,\omega_n = 35 \text{ [m]}$$

$$A_{máx} = A\,\omega_n^2 = 245 \text{ [m/s}^2\text{]}$$



$T = \pi/50$ $\quad f = \frac{50}{\pi}$ [Hz]

$T = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{T} = 100$

$100 = \sqrt{\frac{k}{m}}$

$100^2 = \frac{k}{m}$

$k = 0.1\,(100)^2 = 1000$ N/m

$0.2 = \frac{1}{2} m v^2$

$V_{máx} = 2$ m/s

$V_{máx} = A\,\omega_n$

$A = 0.02$

$x = A \cos(\omega t + \varphi)$

$x = 0.02 \cos(100t + \varphi)$

$x = 0.02 \cos\left(100t + \frac{\pi}{6}\right)$

$\theta = \cos^{-1}\left(\frac{0.01}{0.02}\right)$

$\theta = 60° = \frac{\pi}{3}$

Trabajo y Energia

Tuesday, February 5, 2019 6:52 AM

Trabajo y Energía

Objetivo: movimiento de la partícula, conceptos de trabajo y Energía

Introducción

Trabajo [T]

Escalar

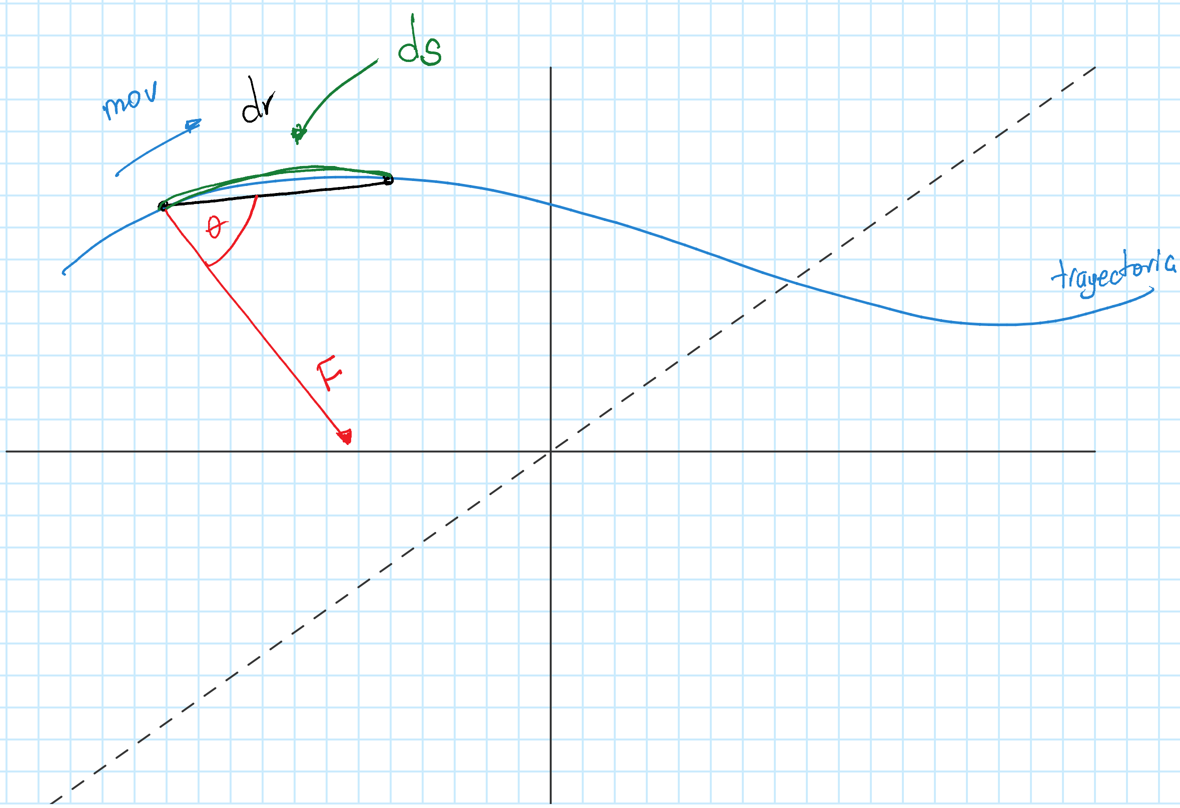
Permite cuantificar la transferencia de energía o la interconversión de energía.

- Trabajo Mecánico Te
  fuerza y desplazamiento

Energia [E]

Capacidad, aptitud o propiedad de la materia para realizar un trabajo

Trabajo mecánico



plano osculador

- velocidad
- aceleración

$ds = |dr|$

$d\vec{T} = F\, d\vec{r}$

$= F\, dr \cos(\theta)$

$= F_T\, ds$

signo

$= F_T\, ds \begin{cases} (+) & \text{activo} \\ (-) & \text{resistivo} \end{cases}$

$d\vec{T} = \vec{dr} \cdot F$

$\vec{F} = \vec{Fi} + \vec{Fj} + \vec{Fk}$

$\vec{dr} = d\vec{xi} + dyj + d\vec{zk}$

$dT = F_x dx + F_y dy + F_z dz$